

# **Multiphase Flow and Heat Transfer**

- INTRODUCTION -

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# Multiphase Flow and Heat Transfer

## Prerequisites

Heat and Mass Transfer

## Course

Mon: 10 AM to 11 AM, Room - 105

Wed: 4 AM to 5 PM, Room - 105

Thu: 11 AM to 12 PM, Room - 105

## Weblinks

[www.iitp.ac.in/~sudheer/ME546/](http://www.iitp.ac.in/~sudheer/ME546/)

# References

*Liquid-Vapor Phase-Change Phenomena*, Van P. Carey, Taylor and Francis: 2<sup>nd</sup> Edition, 2007.

*Convective Boiling and Condensation*, John G. Collier and John R. Thome, Oxford Science: 3<sup>rd</sup> Edition, 1996.

*Fundamentals of Heat and Mass Transfer*, Frank P. Incropera *et al.*, Wiley: 6<sup>th</sup> Edition, 2013.

*Two-Phase Flow and Heat Transfer*, P.B. Whalley, Oxford University Press, 1996.

# Basics

## Phase:

Macroscopic state of matter which is homogeneous in chemical composition and physical structure. Gas, Liquid & solid

### Gas-Liquid

Steam and water; Air and water

### Liquid-Solid

Plasma and platelets (Blood)

### Liquid-Liquid

Oil and water

# Daily examples

- Dust storms
- Rain, Clouds, snow
- Ocean waves
- Geysers, boiling water
- Inkjets
- Tire splash
- Sloshing



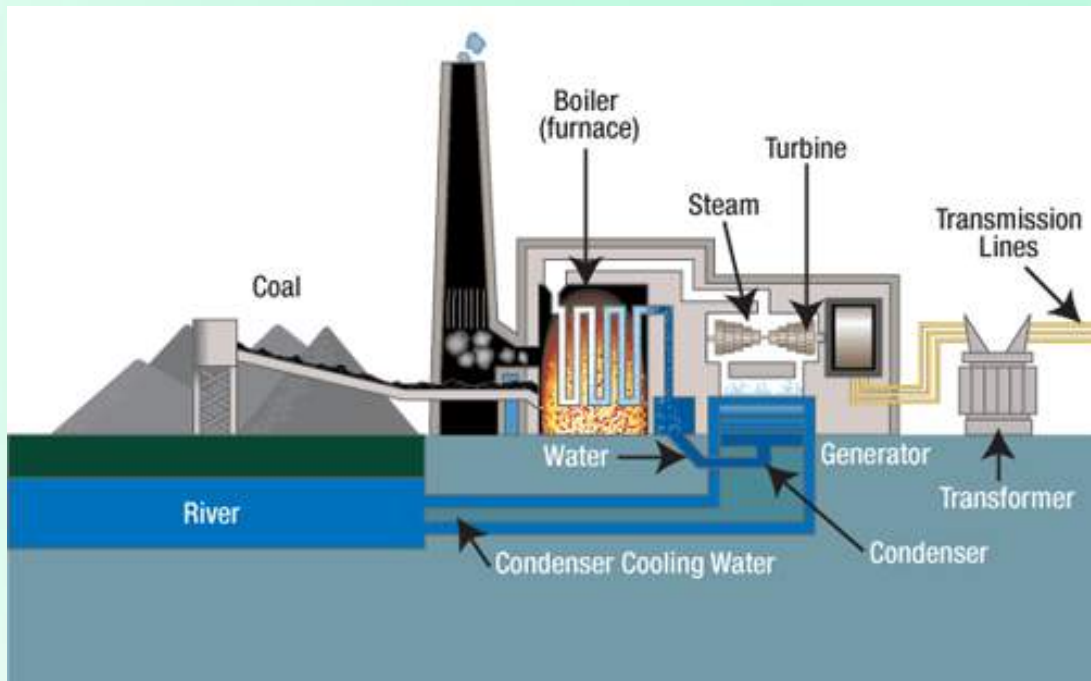
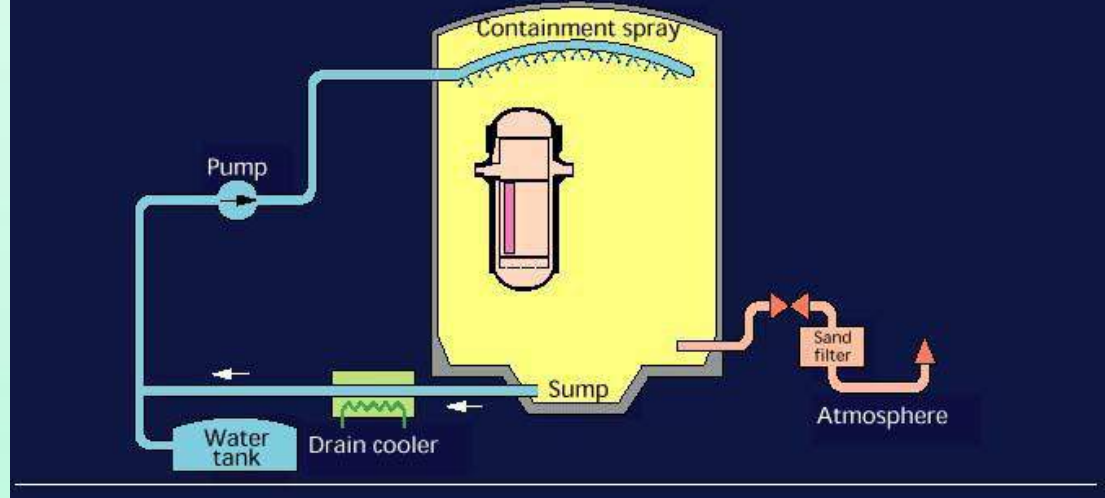
# Industrial applications

- Steam generators and condensers
- steam turbines (power plants)
- Coal fired furnaces
- Refrigeration
- Liquid sprays
- Cryogenics
- Material processing - Quenching
- Process plants – Chemical/petroleum
- Electronic component cooling
- Medical applications

# Spray cooling

## Emergency shut down of nuclear power plant

### CONTAINMENT SPRAY SYSTEM

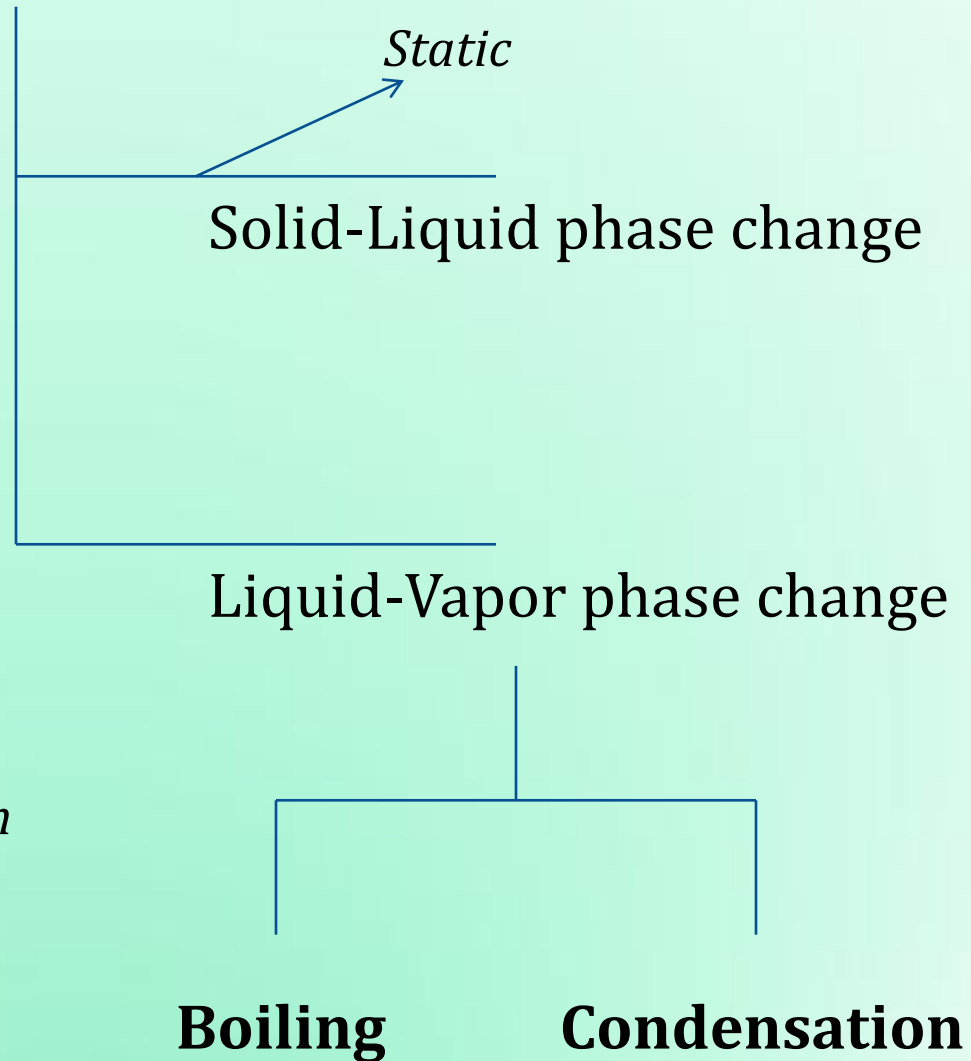


## Steam power plant

# Phase change

Melting and Solidification →

No motion



*Continuous, also energy equation*

# Boiling and Condensation

## Thermodynamics

When the temperature of a liquid at a specified pressure is raised to the saturation temperature  $T_{sat}$  at that pressure, **Boiling** occurs.

Liquid-Vapor transformation:  $T_s > T_{sat}$  at a given pressure

When the temperature of a vapor is lowered to  $T_{sat}$ , **Condensation** occurs.

Vapor-Liquid transformation:  $T_s < T_{sat}$  at a given pressure

# Typical values of convection heat transfer coefficient

Process	$h$ (W/m <sup>2</sup> .K)
Free convection	Gases 2-25
	Liquids 50-1000
Forced convection	Gases 25-250
	Liquids 50-20,000
Convection with phase change Boiling and Condensation	2500-100,000

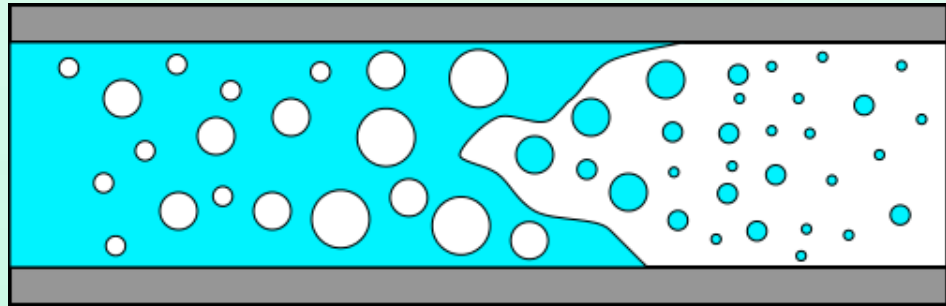
## Convection during phase change depends on

Latent heat of vaporization of the fluid,  $h_{fg}$

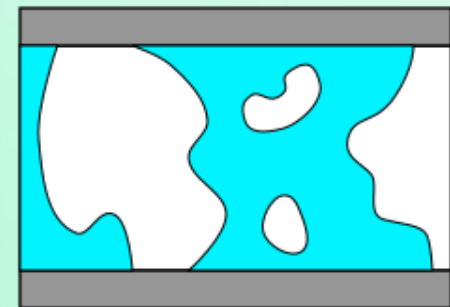
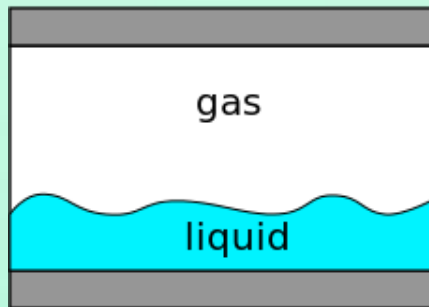
Surface tension,  $\sigma$ , at the liquid-vapor interface

Properties of the fluid in each phase

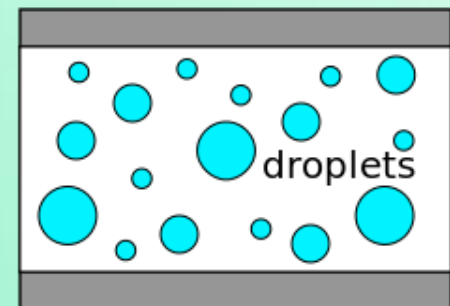
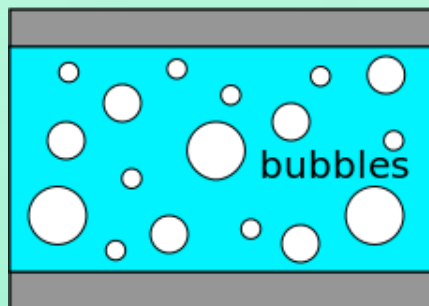
# Examples



a) Transient two-phase flow.



b) Separated two-phase flow.



c) Dispersed two-phase flow.

# **Multiphase Flow and Heat Transfer**

- Boiling Introduction -

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# Boiling

$$Q_{boiling} = h (T_s - T_{sat}) \text{ W/m}^2$$

## Free and Forced convection depends on

density, specific heat, viscosity and thermal conductivity of the fluid

## Boiling Heat Transfer depends on

density, specific heat, viscosity and thermal conductivity of liquid

Latent heat of vaporization

Surface tension at the liquid-vapor interface

# Boiling and Evaporation

## Boiling

The process of addition of heat to a liquid such a way that generation of vapor occurs.

Solid-liquid interface

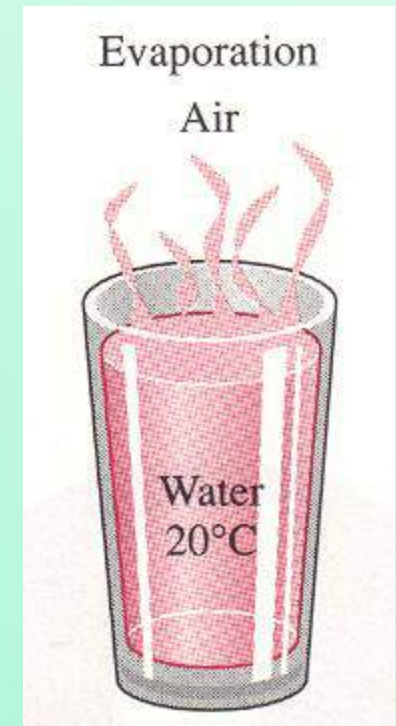
Characterized by the rapid formation of vapor bubbles

## Evaporation

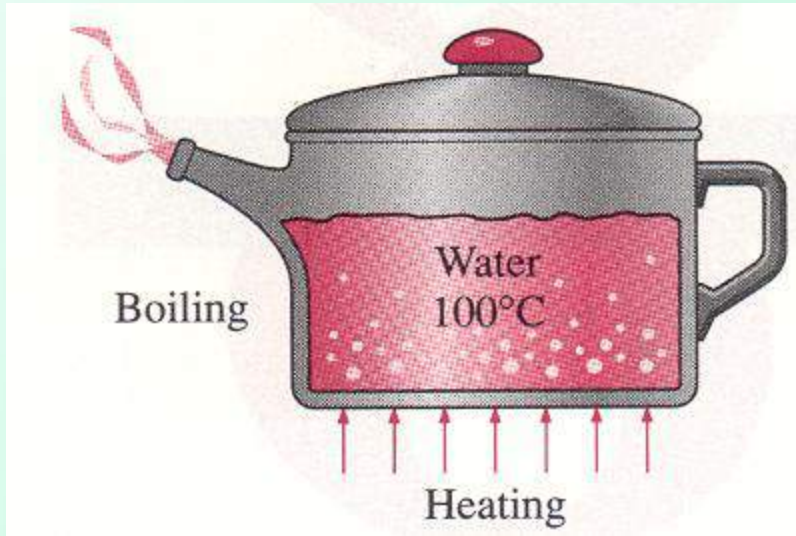
Liquid-vapor interface

$P_v < P_{sat}$  of the liquid at a given temp

No bubble formation or bubble motion

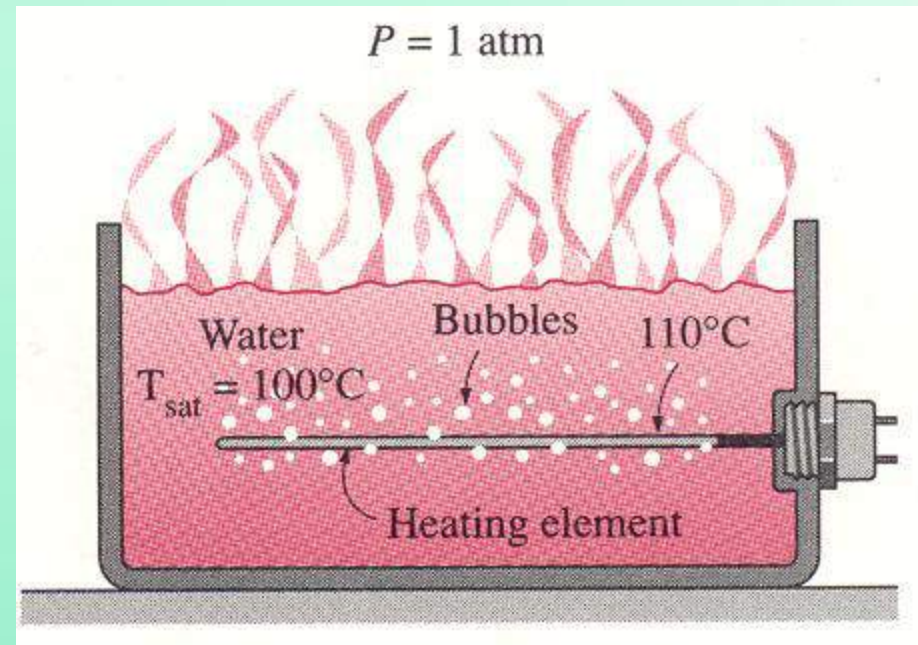


# Boiling



Boiling occurs at a solid-liquid interface

Boiling occurs when a liquid is brought into contact with a surface at a temperature above the saturation temperature of the liquid



# Bubble

- The boiling processes in practice **do not occur under equilibrium** conditions.
- Bubbles exist because of the surface tension at the liquid vapor interface due to the attraction force on molecules at the interface toward the liquid phase.
- The temperature and pressure of the vapor in a bubble are usually different than those of the liquid.
- Surface tension  $\downarrow$   $\uparrow$  Temperature
- Surface tension = 0 at critical temperature
- No bubbles at supercritical pressures and temperatures

# Classification - Boiling

**Pool Boiling**

**Flow Boiling**

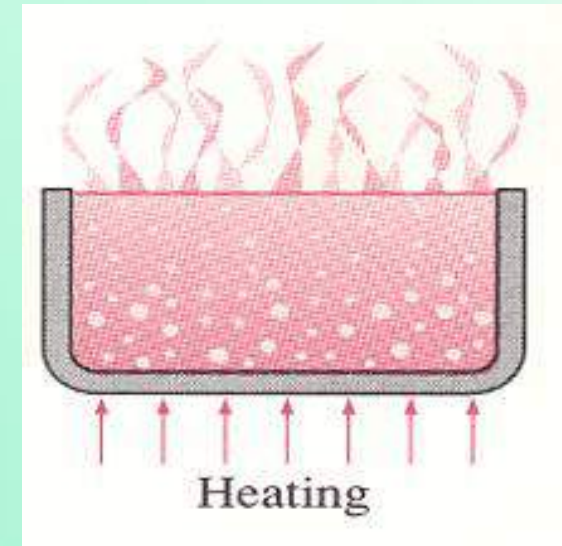
**(Forced Convection Boiling)**

## **Pool Boiling**

The fluid is stationary

Motion of the fluid is due to natural convection currents

Motion of the bubbles under the influence of buoyancy.



**Pool Boiling**

# Classification - Boiling

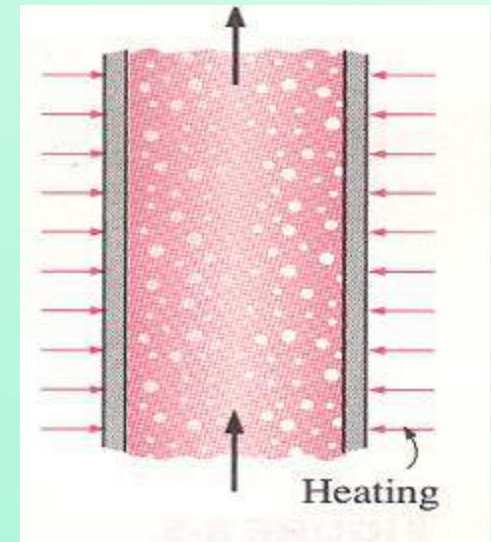
**Pool Boiling**

**Flow Boiling**  
**(Forced Convection Boiling)**

## **Flow Boiling**

Fluid is forced to move in a heated pipe or surface by external means such as pump.

Flow boiling is always accompanied by other convection effects.

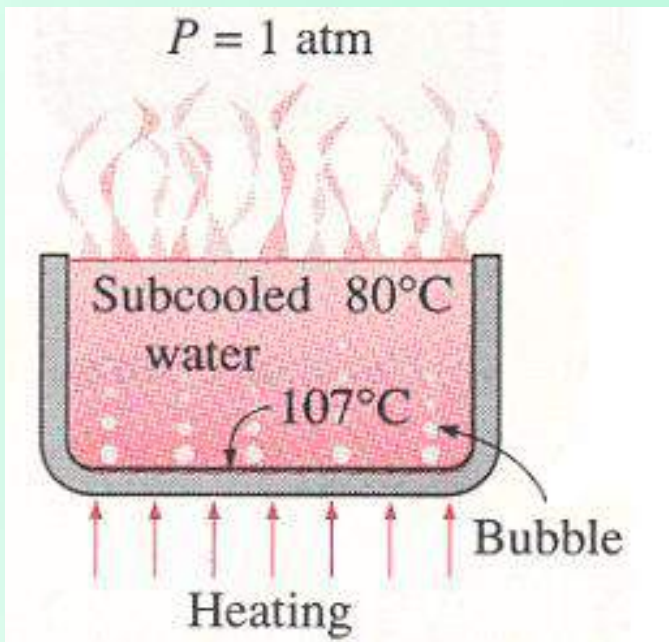


**Flow Boiling**

# Classification - Boiling

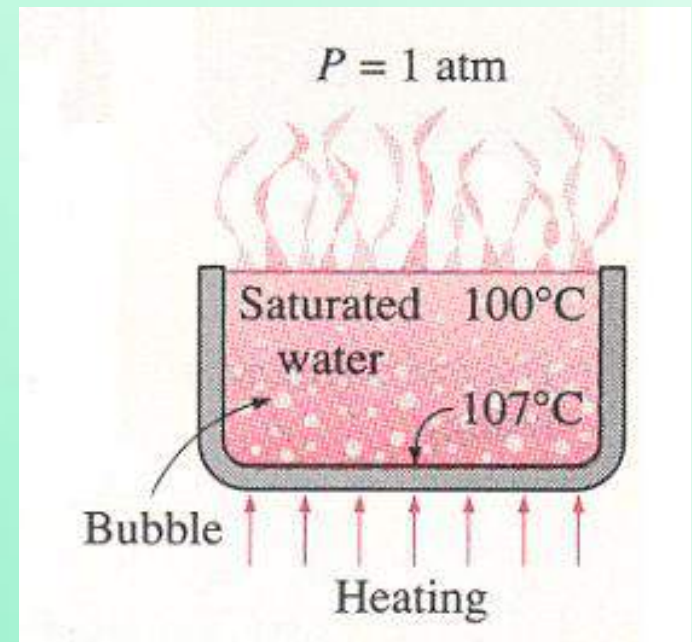
## Subcooled Boiling

$$T_{\text{bulk of liquid}} < T_{\text{sat}}$$



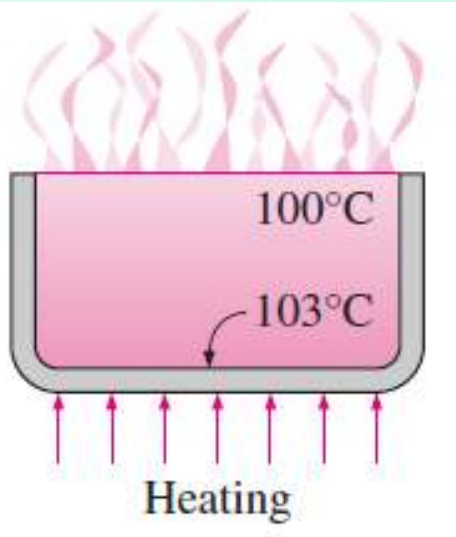
## Saturated Boiling

$$T_{\text{bulk of liquid}} = T_{\text{sat}}$$

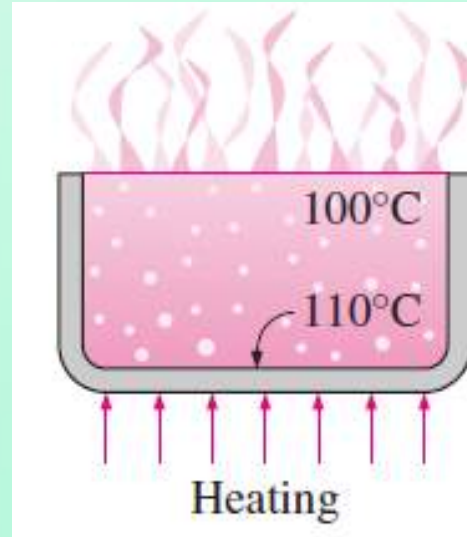


# Boiling Regimes – Nukiyama, 1934

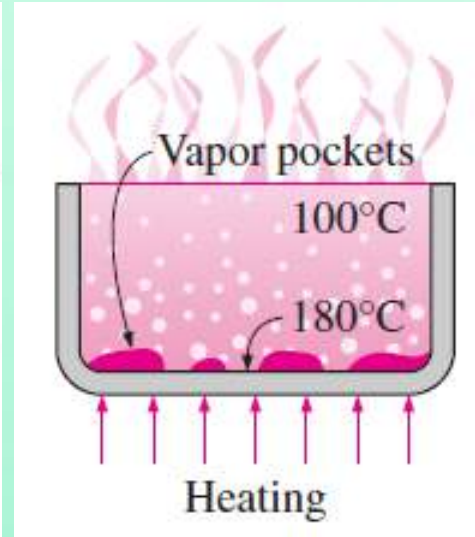
## Nukiyama – 1934, Boiling Regimes



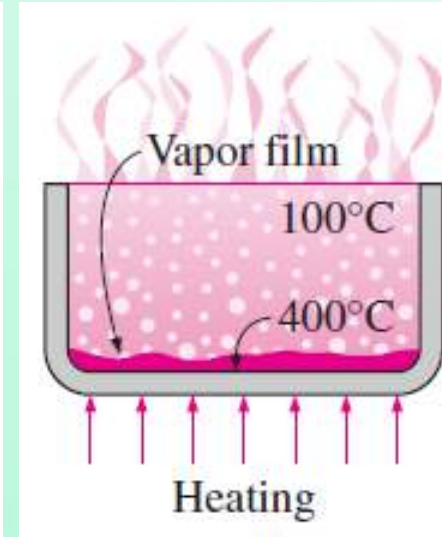
**Natural convection boiling**



**Nucleate boiling**

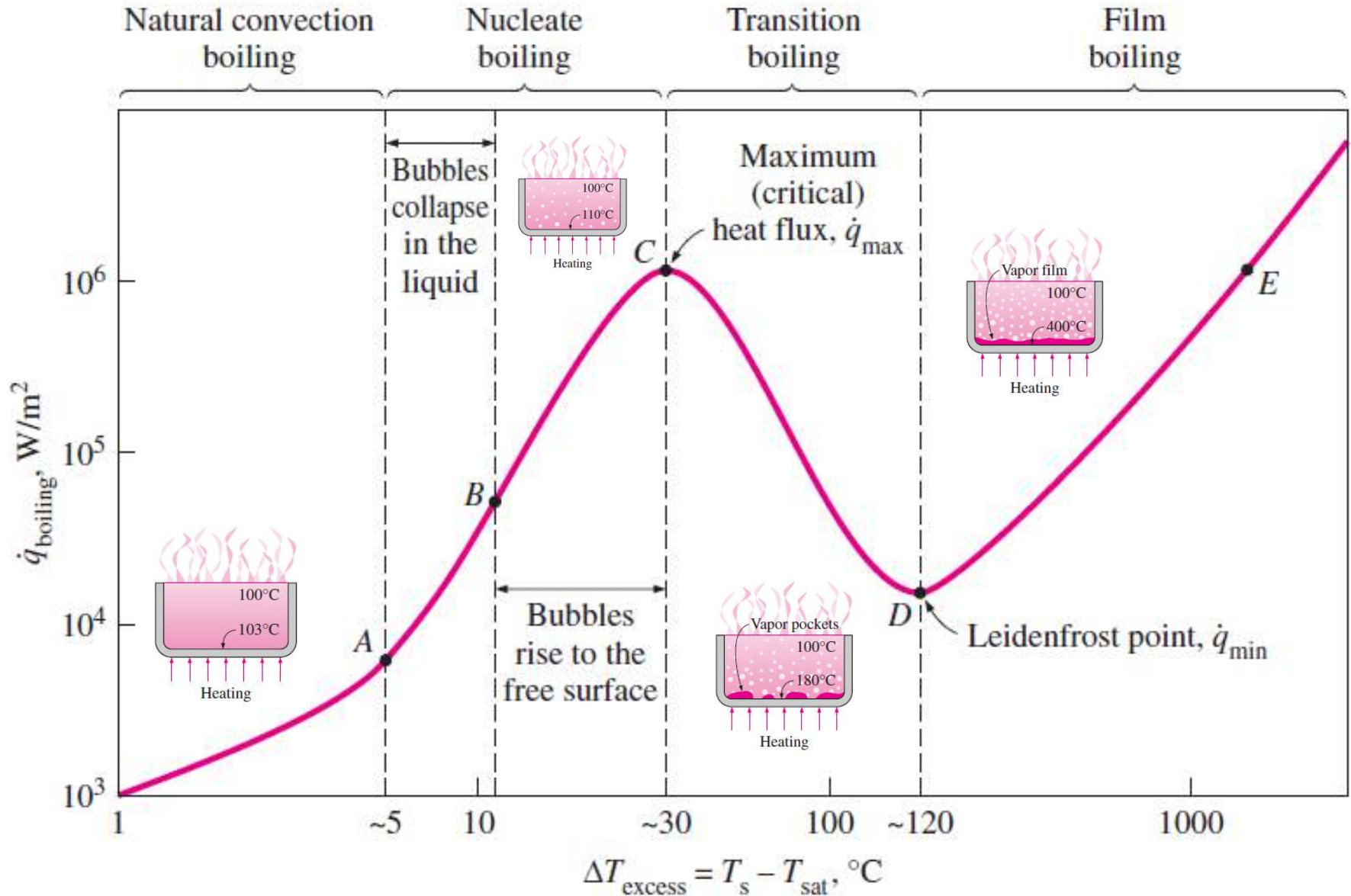


**Transition boiling**



**Film boiling**

# Nukiyama's boiling curve for saturated water at atmospheric pressure



# Boiling regimes – Methanol on horizontal 1 cm steam-heated copper tube



1. Natural convection



Onset of boiling



2. Individual bubble regime



3. Regime of slugs and bubbles



4. Transition film boiling

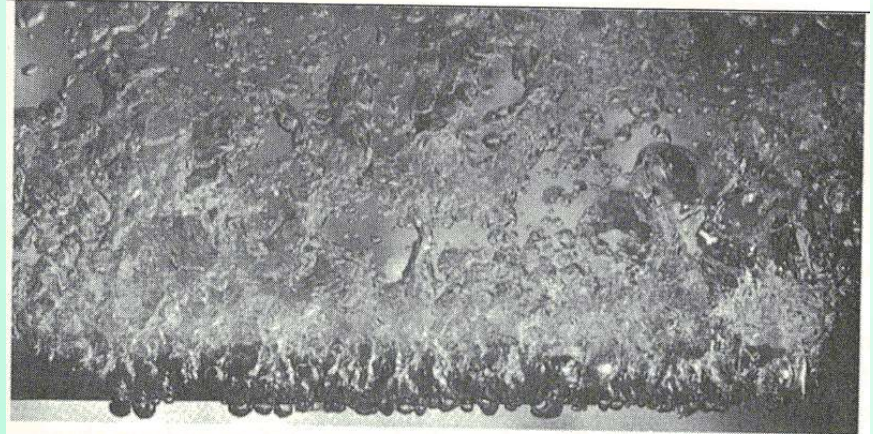


5. Stable film boiling

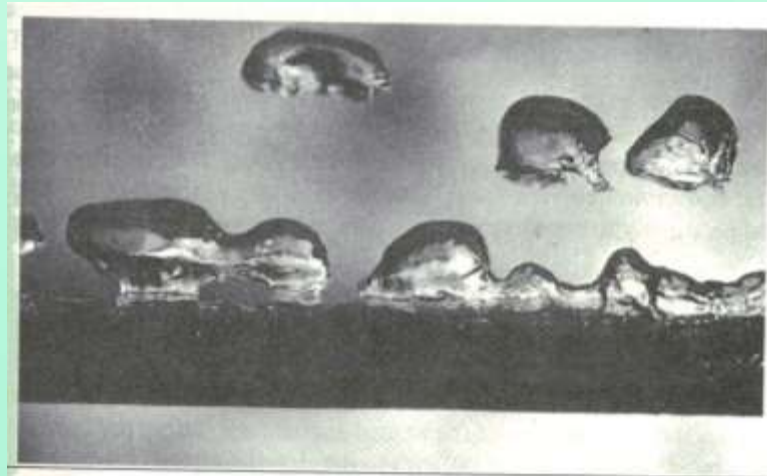
# Boiling regimes – Methanol on horizontal 1 cm steam-heated copper tube



**Nucleate boiling**



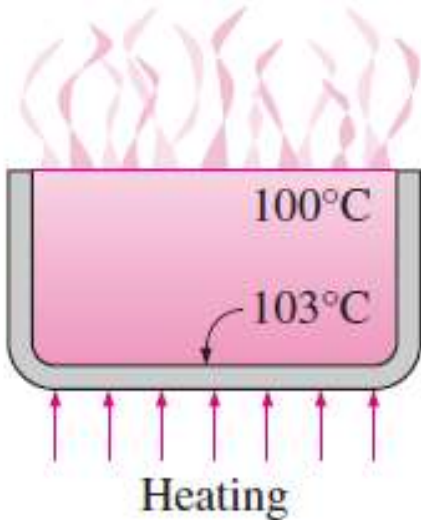
**Flow boiling**



**Film boiling**

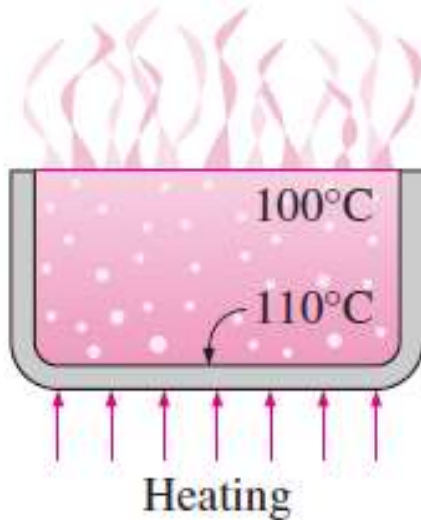
## Natural convection Boiling

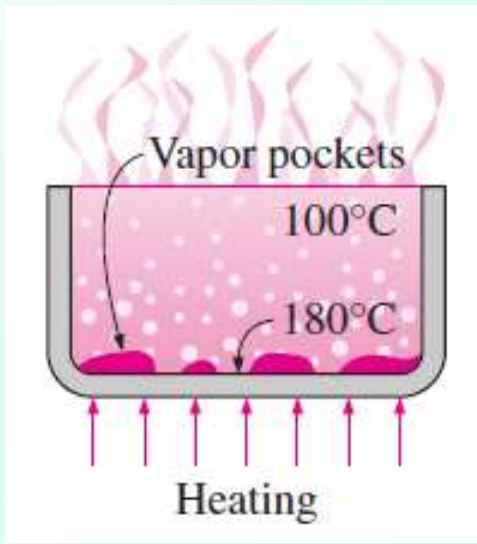
- Governed by natural convection currents.
- Heat transfer from the heating surface to the fluid is by natural convection.



## Nucleate Boiling

- The stirring and agitation caused by the entrainment of the liquid to the heater surface is primarily responsible for the increased heat transfer coefficient and heat flux in the region of nucleate boiling.
- High heat transfer rates are achieved in nucleate boiling.

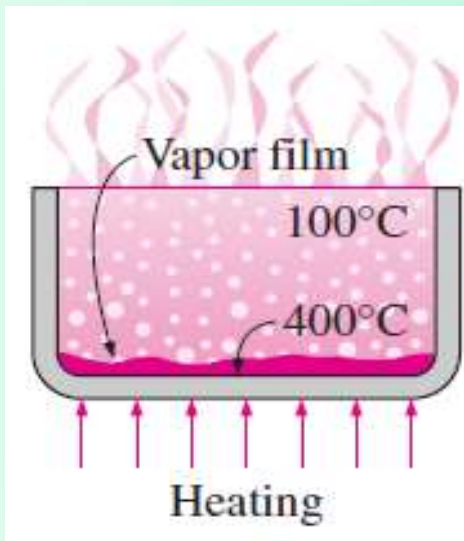




## Transition Boiling (Unstable film boiling)

- Heat flux decreases because of larger fraction of heater surface is covered by a vapor film which acts as a insulation because of the low thermal conductivity of the vapor relative to the liquid.

## Film Boiling

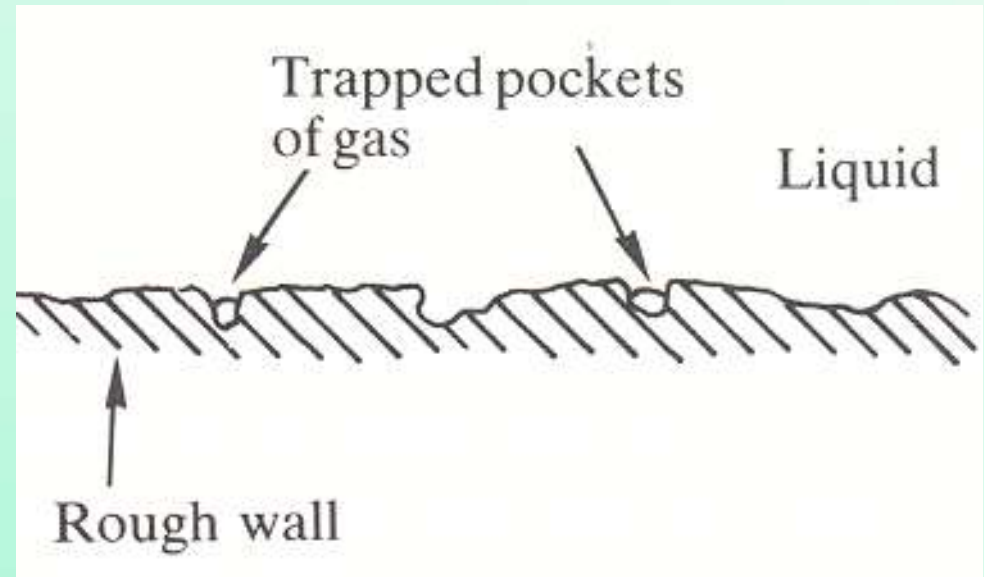


- The presence of a vapor film between the heater surface and the liquid is responsible for the low heat transfer rates
- Heat transfer rate increases with increasing excess temperature as a result of heat transfer from the heated surface to the liquid through the vapor film by radiation.

# Bubble Nucleation - Boiling Inception

The process of bubble formation is called **Nucleation**

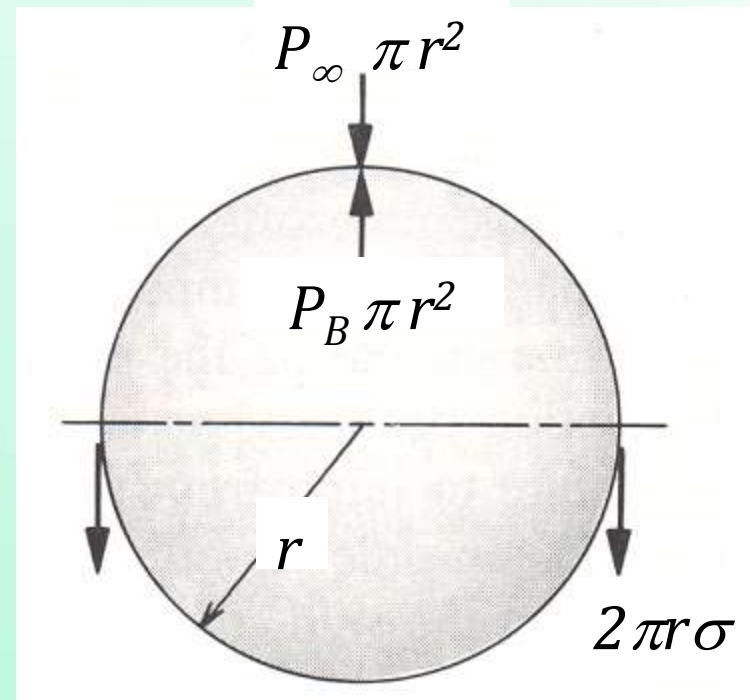
Enlarged view of a boiling surface



- The cracks and crevices do not constitute nucleation sites for the bubbles. Must contain pockets of gas/air trapped.
- It is from these pockets of trapped air that the vapor bubbles begin to grow during nucleate boiling.
- These cavities are the sites at which bubble nucleation occurs.

When a liquid contacts the surface, surface tension forces prevent the liquid from entering the smaller cavities in which air or other gases are trapped.

- $r$  radius of the bubble
- $\sigma$  surface tension
- $P_B$  pressure inside the bubble
- $P_\infty$  pressure in the liquid or the ambient pressure

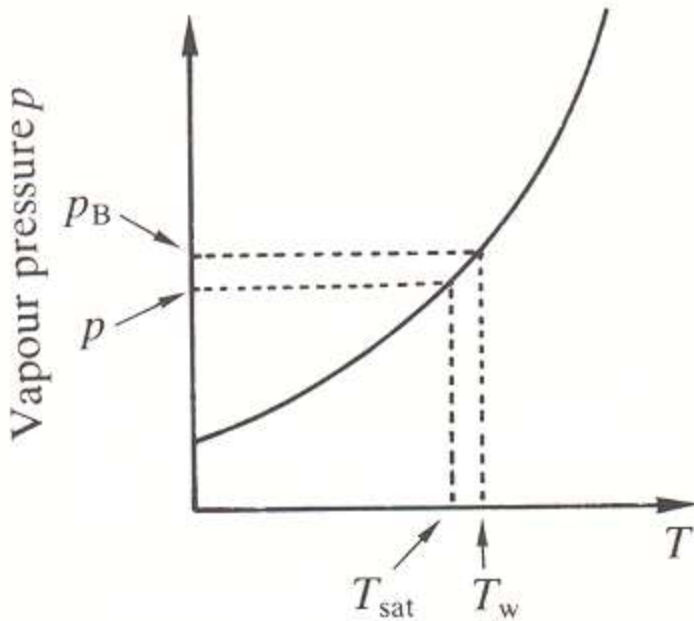


For static equilibrium, the surface tension force balances the net pressure force:

$$2\pi r \sigma = (P_B - P_\infty) \pi r^2$$

$$P_B = P_\infty + \frac{2\sigma}{r}$$

Young-Laplace equation



$P_B$  is maximum when  $r = R$  (the cavity radius)

The wall temperature  $T_w$  must be high enough to vaporize the liquid at a pressure of  $P_B$

For the bubble to grow, the required condition:

Vapor pressure curve:  
superheat required for nucleation

$$T_w > T_{sat} + \frac{dT}{dp} (P_B - P_\infty)$$

Slope of the vapor pressure curve found from the Clausius-Clapeyron eq.

$$\frac{dp}{dT} = \frac{h_{lv}}{(v_v - v_l)T_{sat}}$$

$h_{lv}$   
 $T_{sat}$   
 $v_v$   
 $v_l$

latent heat of vaporization  
saturation temperature  
specific volume of the gas  
specific volume of the liquid

Then, if  $v_v \gg v_l$  and, since  $v_v = 1/\rho_v$

$$\frac{dp}{dT} = \frac{h_{lv}}{(v_v - v_l) T_{sat}} \Rightarrow \frac{dT}{dP} = \frac{T_{sat}}{\rho_v h_{lv}}$$

$$T_w > T_{sat} + \frac{dT}{dp} (P_B - P_\infty) \Rightarrow T_w > T_{sat} + \frac{T_{sat}}{\rho_v h_{lv}} \frac{2\sigma}{R}$$

If  $\Delta T_{sat}$  is the value of  $(T_w - T_{sat})$  at which nucleation starts, then the cavity radius is given by

$$R = \frac{2\sigma T_{sat}}{\rho_v h_{lv} \Delta T_{sat}}$$

$$\Delta T_{sat} = T_w - T_{sat}$$

For water at 1 bar

$$T_{sat} = 373 \text{ K}$$

$$\sigma = 0.059 \text{ N/m}$$

$$h_{fv} = 2.256 \times 10^6 \text{ J/kg}$$

$$\rho_v = 0.598 \text{ kg/m}^3$$

# Critical radius for Nucleation

For water at 1 bar, is commonly about 5 K so, given

$$T_{sat} = 373 \text{ K}; \sigma = 0.059 \text{ N/m}; h_{lv} = 2.256 \times 10^6 \text{ J/kg}; \rho_v = 0.598 \text{ kg/m}^3$$

**R** is found to be about 6.5  $\mu\text{m}$ , and typically cavity sizes are in the micron range. If the cavity size is known, then clearly the wall superheat required to start nucleate boiling can be calculated.

Real surfaces, of course, can contain a range of cavity sizes. As the wall superheat is increased, cavities of smaller and smaller radius are able to become active and initiate nucleation.

Minimum size of active nucleation sites on smooth metallic surfaces;

Water  $\sim 5 \mu\text{m}$

Organics and refrigerants  $\sim 0.5 \mu\text{m}$

Cryogenic fluids on aluminum or copper  $\sim 0.1\text{-}0.3\mu\text{m}$

# Heat Transfer in Nucleate Boiling

**Rohsenow** postulated:

- Heat flows from the surface first to the adjacent liquid, as in any single-phase convection process.
- The high heat transfer coefficient associated with nucleate boiling is a result of local agitation due to liquid flowing behind the wake of departing bubbles.

Thus, it may be possible to adapt a single-phase forced convection heat transfer correlation to nucleate pool boiling, if we could specify the appropriate length and velocity scales associated with the convection process.

# Heat Transfer in Nucleate Boiling

$$Nu = f(Re, Pr)$$

$$Nu = \frac{hL_b}{k_l}; \quad Re = \frac{\rho_v u_b L_b}{\mu_l}; \quad Pr = \frac{\mu_l C_{pl}}{k_l}$$

Velocity is taken as the liquid velocity in towards the surface which is to supply the vapor which is being produced, so:

$$u_b \sim \frac{q''}{h_{lv} \rho_v}$$

Length scale is taken to be, 
$$L_b \sim \left[ \frac{\sigma}{g(\rho_l - \rho_v)} \right]^{\frac{1}{2}}$$

# Heat Transfer in Nucleate Boiling

$$Nu = \frac{hL_b}{k_l} = \frac{h}{k_l} \left[ \frac{\sigma}{g(\rho_l - \rho_v)} \right]^{\frac{1}{2}}$$

$$Pr = \frac{\mu_l C_{pl}}{k_l}$$

$$Re = \frac{\rho_v u_b L_b}{\mu_l} = \frac{\rho_v}{\mu_l} \frac{q''}{h_{lv} \rho_v} \left[ \frac{\sigma}{g(\rho_l - \rho_v)} \right]^{\frac{1}{2}}$$

$$Nu = \frac{1}{C_{sf}} Re^{1-m} Pr^{-n}$$

$$h = \frac{q''}{T_w - T_{sat}}$$

# Rohsenow Correlation

$$\frac{C_{pl} (T_w - T_{sat})}{h_{lv}} = C_{sf} \left[ \frac{q''}{\mu_l h_{lv}} \sqrt{\frac{\sigma}{g(\rho_l - \rho_v)}} \right]^m \left[ \frac{C_{pl} \mu_l}{k_l} \right]^{1+n}$$

$$\frac{q''}{\mu_l h_{lv}} \sqrt{\frac{\sigma}{g(\rho_l - \rho_v)}} = \left( \frac{1}{C_{sf}} \right)^{\frac{1}{m}} \left[ \frac{C_{pl} (T_w - T_{sat})}{h_{lv} \text{Pr}} \right]^{\frac{1}{m}} \left[ \frac{C_{pl} \mu_l}{k_l} \right]^{-\frac{n}{m}}$$

$m = 0.33$  and  $1 + n = 1$  for water and  $1.7$  for other fluids

$C_{st}$  is the surface-fluid constant. Depends on both the surface and the fluid.  
Typical range:  $0.0025$  and  $0.015$

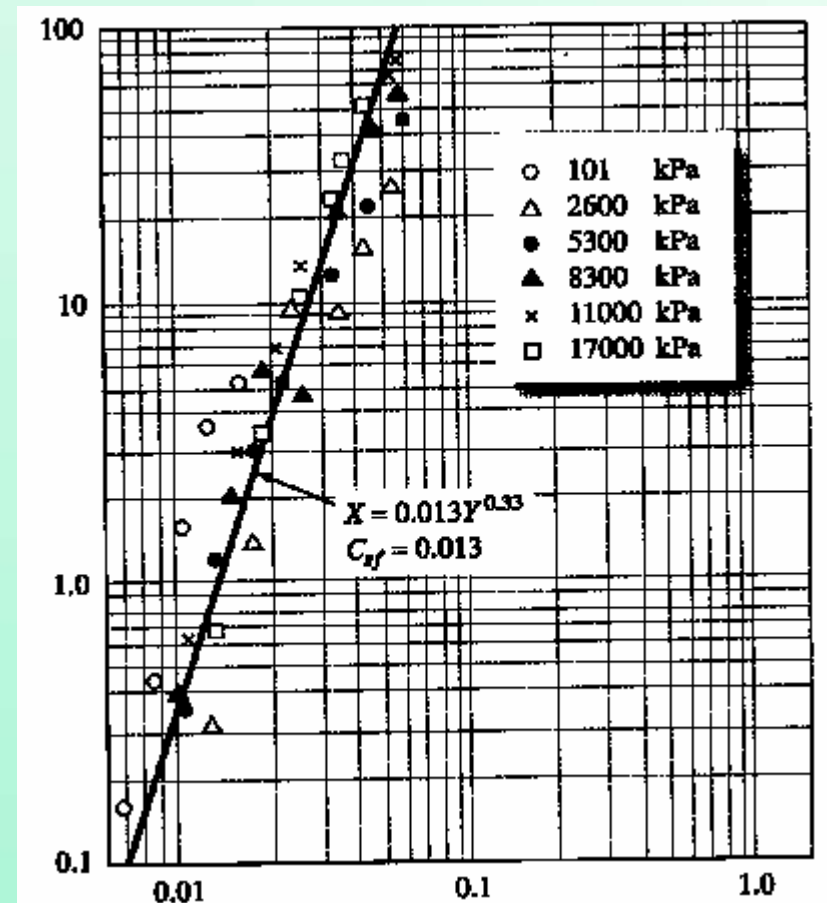
For a given  $\Delta T_{sat}$ , the heat flux is proportional to  $(C_{st})^{-3}$ . Since  $C_{st}$  can vary by a factor of  $10$ , the heat flux can vary by a factor of  $1000$ .

**Applicable only for clean surfaces**

# Rohsenow Correlation

$$\left[ \frac{q''}{\mu_f h_{fg}} \sqrt{\frac{\sigma}{g(\rho_f - \rho_g)}} \right]$$

Correlation of pool boiling heat transfer data for Platinum wire-water by the method of Rohsenow



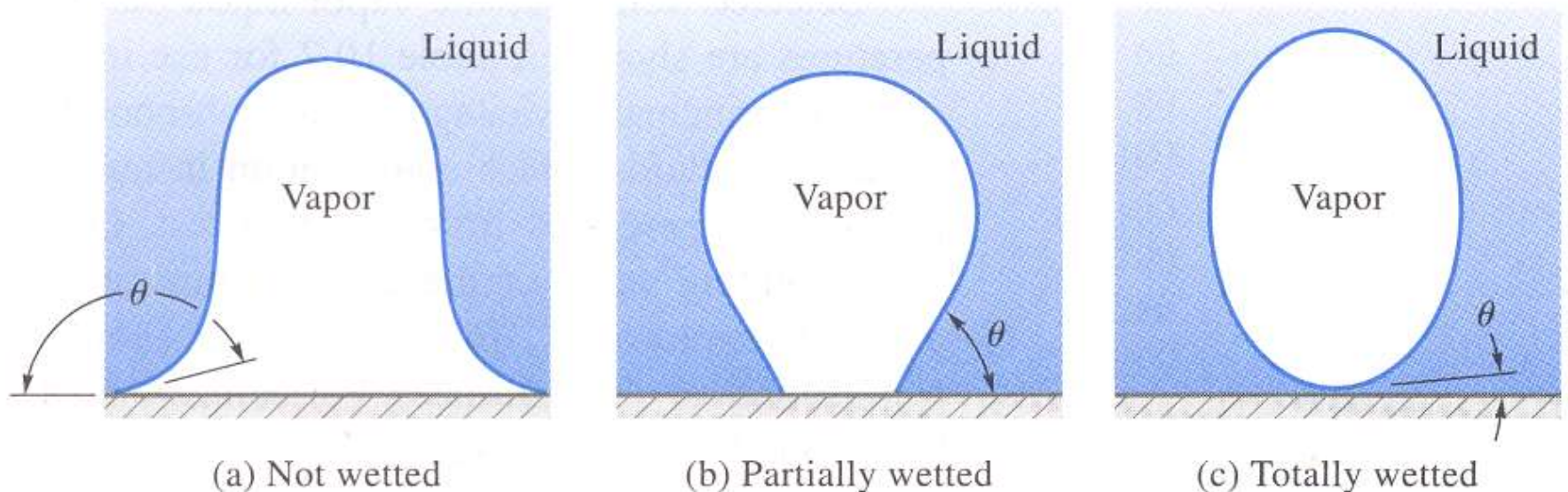
$$\frac{C_{pf} (T_w - T_{sat})}{h_{fg} Pr}$$

The most important variables affecting  $C_{sf}$  are

- Surface roughness of the heater which determines the number of nucleation sites at a given temperature.
- Angle of contact between the bubble and heating surface is a measure of wettability of a surface with a particular fluid.

Surface is **hydrophilic** (Wetted), if  $\theta < 90^\circ$

Surface is **hydrophobic** (Not wetted), if  $\theta > 90^\circ$



Effect of surface Wettability on the bubble contact angle,  $\theta$

## Coefficient ( $C_{sf}$ ) in Rohsenow Correlation

For contaminated surfaces, the exponent of Prandtl number of liquid is found to vary between 0.8 and 2.0.

Contamination also affects the other exponent in the equation and  $C_{sf}$

If no data is available, then consider,  $C_{sf} = 0.013$

**TABLE 10.1** Values of the coefficient  $C_{sf}$  in Eq. (10.2) for various liquid-surface combinations

Fluid-heating Surface Combination	$C_{sf}$
Water on scored copper [16] <sup>a</sup>	0.0068
Water on emery-polished copper [16]	0.0128
Water-copper [21]	0.0130
Water on emery-polished, paraffin-treated copper [16]	0.0147
Water-brass [23]	0.0060
Water on Teflon coated stainless steel [16]	0.0058
Water on ground and polished stainless steel [16]	0.0080
Water on chemically etched stainless steel [16]	0.0133
Water on mechanically polished stainless steel [16]	0.0132
Water-platinum [17]	0.0130
<i>n</i> -Pentane on lapped copper [16]	0.0049
<i>n</i> -Pentane on emery-rubbed copper [16]	0.0074
<i>n</i> -Pentane on emery-polished copper [16]	0.0154
<i>n</i> -Pentane on emery-polished nickel [16]	0.0127
<i>n</i> -Pentane-chromium [22]	0.0150
Isopropyl alcohol-copper [21]	0.00225
<i>n</i> -Butyl alcohol-copper [21]	0.00305
Ethyl alcohol-chromium [22]	0.0027
Carbon tetrachloride on emery-polished copper [16]	0.0070
Carbon tetrachloride-copper [21]	0.0130
Benzene-chromium [22]	0.0100
50% $K_2CO_3$ -copper [21]	0.00275
35% $K_2CO_3$ -copper [21]	0.0054

# Heat Transfer in Nucleate Boiling

Another frequently quoted **Forster-Zuber correlation**

$$q'' = 0.00122 \left( \frac{k_f^{0.79} C_{pf}^{0.45} \rho_f^{0.49}}{\sigma^{0.5} \mu_f^{0.29} h_{fg}^{0.24} \rho_g^{0.24}} \right) [T_w - T_{sat}(P_f)]^{1.24} \Delta P_{sat}^{0.75}$$

$\Delta P_{sat}$  is the difference in saturation pressure corresponding to a difference in saturation temperature equal to the wall superheat  $T_w - T_{sat}(P_f)$

$k_f$  - kW/m.°C

$C_{pf}$  - kJ/kg.°C

$\rho$  - kg/m<sup>3</sup>

$P$  - Pa

$\sigma$  - N/m

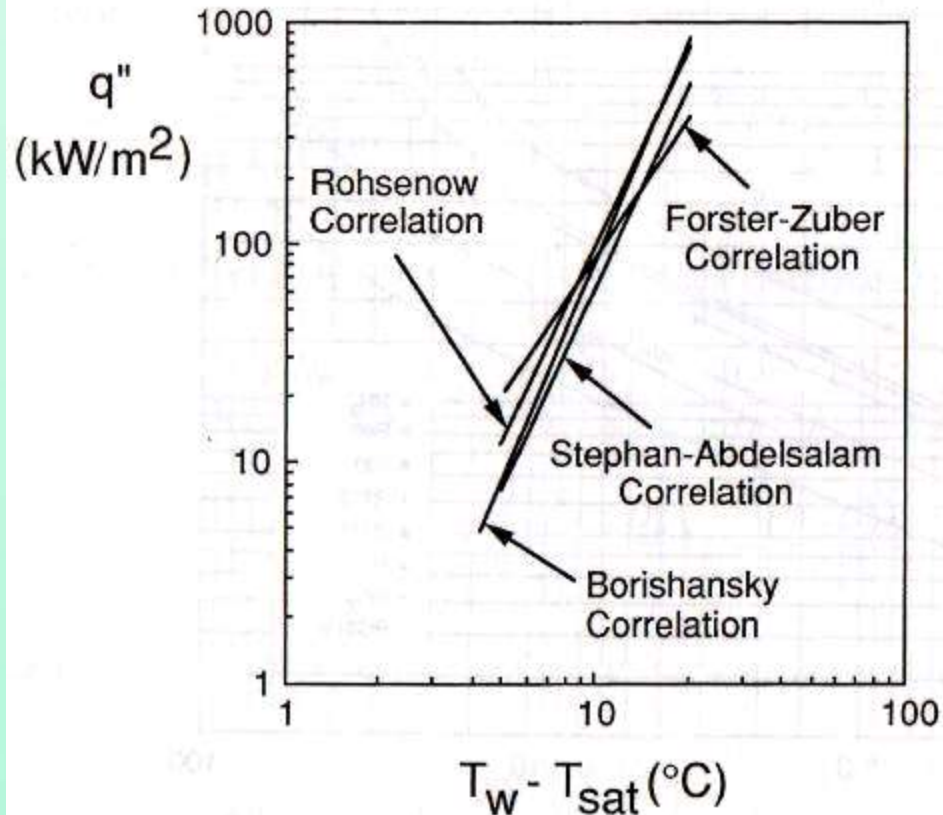
$\mu$  - Ns/m<sup>3</sup>

$h_{fg}$  - kJ/kg

$q''$  - kW/m<sup>2</sup>

# Heat Transfer in Nucleate Boiling

Nucleate Pool Boiling Curves for Water at Atmospheric Pressure



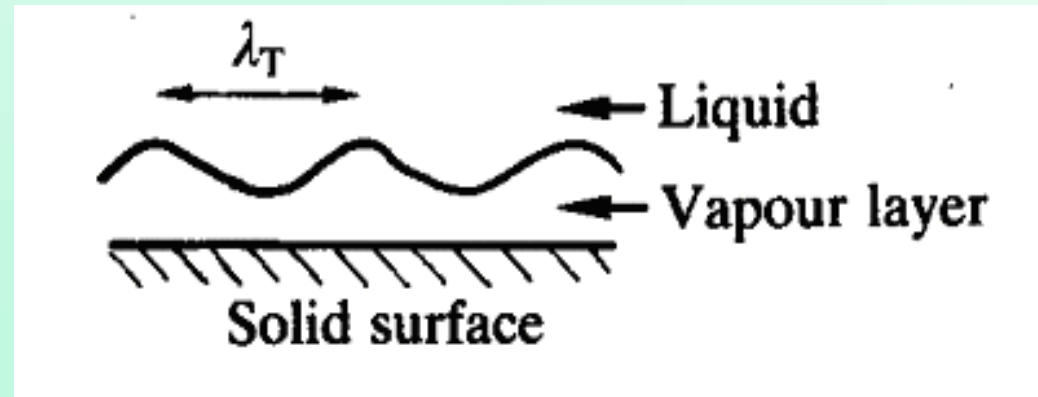
# Critical Heat flux in Nucleate Boiling

- Rohsenow correlation is restricted to nucleate boiling
  - Does not reveal the  $\Delta T_{sat}$  at which the  $q''_{max}$
  - Limiting heat flux: when nucleate boiling breaks down and an insulating vapor film forms
- For a  $q''$  controlled surface, the  $\Delta T_{sat}$  rise after  $q''_{max}$  can be very large (can be  $> 1000$  K)

# Instability of the vapor layer

The light fluid in a layer which has a heavy fluid on top of it is unstable.

The layer breaks down by the formation of waves on its surface as in the figure.



**Taylor waves**

## Rayleigh-Taylor Instability

During boiling, disturbances of all wavelengths are present, there will be some disturbances at small wavelength and long wavelength that will amplify and cause the interface to be unstable.

# Rayleigh-Taylor Instability

Condition for the interface instability of a motionless liquid overlaying a motionless vapor region

$$\omega > \omega_c = \left[ \frac{(\rho_l - \rho_v)g}{\sigma} \right]^{\frac{1}{2}}$$

This condition is called **Rayleigh-Taylor Instability**

Corresponding Critical Wavelength:

$$\lambda_c = \frac{2\pi}{\omega_c} = 2\pi \left[ \frac{\sigma}{g(\rho_l - \rho_v)} \right]^{\frac{1}{2}}$$

The most dangerous wavelength, as they grow most rapidly,  $\lambda_D = \sqrt{3}\lambda_c$

At 1 bar:

$$\sigma = 0.058988 \text{ N/m}$$

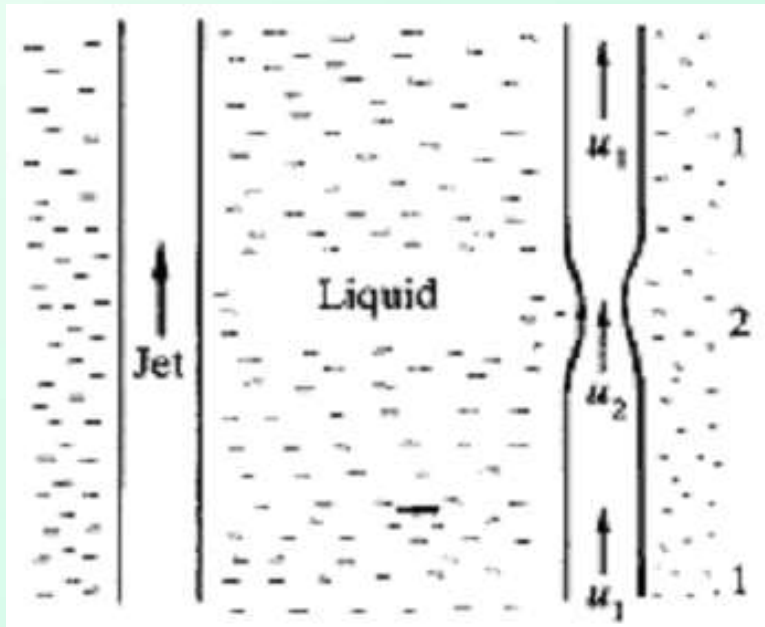
$$\rho_l = 958.63 \text{ kg/m}^3$$

$$\rho_v = 0.59034 \text{ kg/m}^3$$

$$\lambda_c = 15.7 \text{ mm}$$

$$\lambda_D = 27.2 \text{ mm}$$

# Kelvin-Helmholtz instability



A parallel sided jet is not stable.

Consider the random thinning of the jet as illustrated.

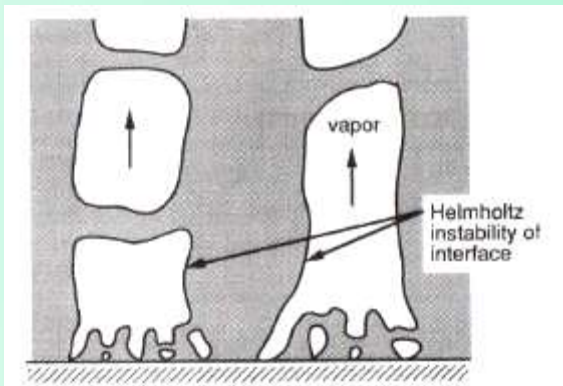
By continuity,  $u_2 > u_1$  and therefore, from Bernoulli's equation  $p_2 < p_1$ .

If the jet is in equilibrium at 1, then the liquid pressure at 2 will push the 'neck' further in and disrupt the jet completely, thus breaking it up. This is a **Kelvin-Helmholtz instability**.

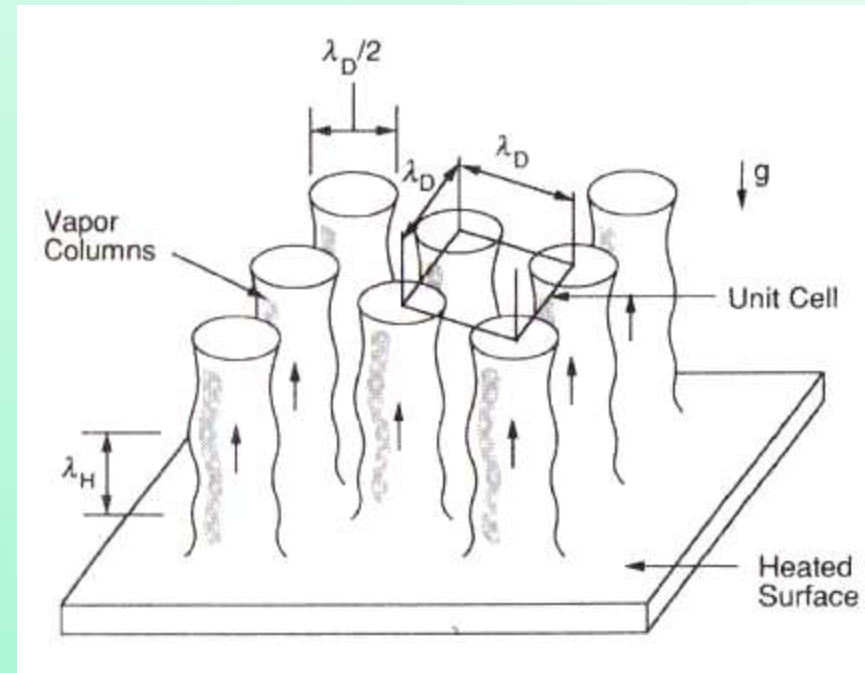
This argument would imply that the vapor jet is always unstable, but the effects of surface tension, which has a stabilizing effect, have been neglected.

# Zuber's model Analysis

1.  $q''_{\max}$  when the interface of columns - Helmholtz unstable.
2. Centerline spacing of columns = most dangerous  $\lambda$  of Taylor instability.
3. Columns dia. = to  $\lambda_D/2$ .
4. The Helmholtz unstable  $\lambda$  imposed on the columns = Taylor wave node spacing  $\lambda_D$ .



Postulated Helmholtz  
instability CHF mechanisms



Vapor column spacing in the  
Zuber critical heat flux model

# Zuber's model Analysis: consider unit cell

Critical Helmholtz velocity for vertical vapor and liquid flow:

$$u_c = |\bar{u}_l - \bar{u}_v| = \left( \frac{\sigma \omega (\rho_l + \rho_v)}{\rho_l \rho_v} \right)^{1/2}$$

Assuming  $\rho_f \ll \rho_g$  and substituting  $\omega = 2\pi/\lambda$ :

$$u_c = \left( \frac{2\pi\sigma}{\rho_v \lambda} \right)^{1/2}$$

Most dangerous wavelength from Taylor instability for liquid in contact with vapor:

$$\lambda_D \sim 2\pi \left[ \frac{3\sigma}{(\rho_l - \rho_v)g} \right]^{1/2}$$

# Zuber's model Analysis: consider unit cell

Critical heat flux, the rate of heat supply to the area  $A_{surf}$

$$q''_{\max} A_{surf} \sim h_{lv} \rho_v u_v A_{col}$$

$$q''_{\max} = \frac{\pi}{16} h_{lv} \rho_v u_v$$

- Since the downward liquid velocity is much smaller than the upward vapor velocity, due to the large density difference between the phases,  $u_c \sim u_v$ .
- The Helmholtz unstable wavelength imposed on the columns is equal to the Taylor wave node spacing  $\lambda_D$ .

$$q''_{\max} = \frac{\pi}{16(3)^{1/4}} h_{fg} \rho_g^{1/2} [\sigma(\rho_f - \rho_g)g]^{1/4}$$

works well for the flat horizontal plates.

# Zuber's Correlation for Critical Heat Flux

The Helmholtz unstable wavelength imposed on the columns is equal to the Taylor wave node spacing  $\lambda_D$ .

$$q''_{\max} = 0.149 h_{lv} \rho_v^{1/2} [\sigma(\rho_l - \rho_v)g]^{1/4}$$

This Eq. works well for the flat horizontal plates.

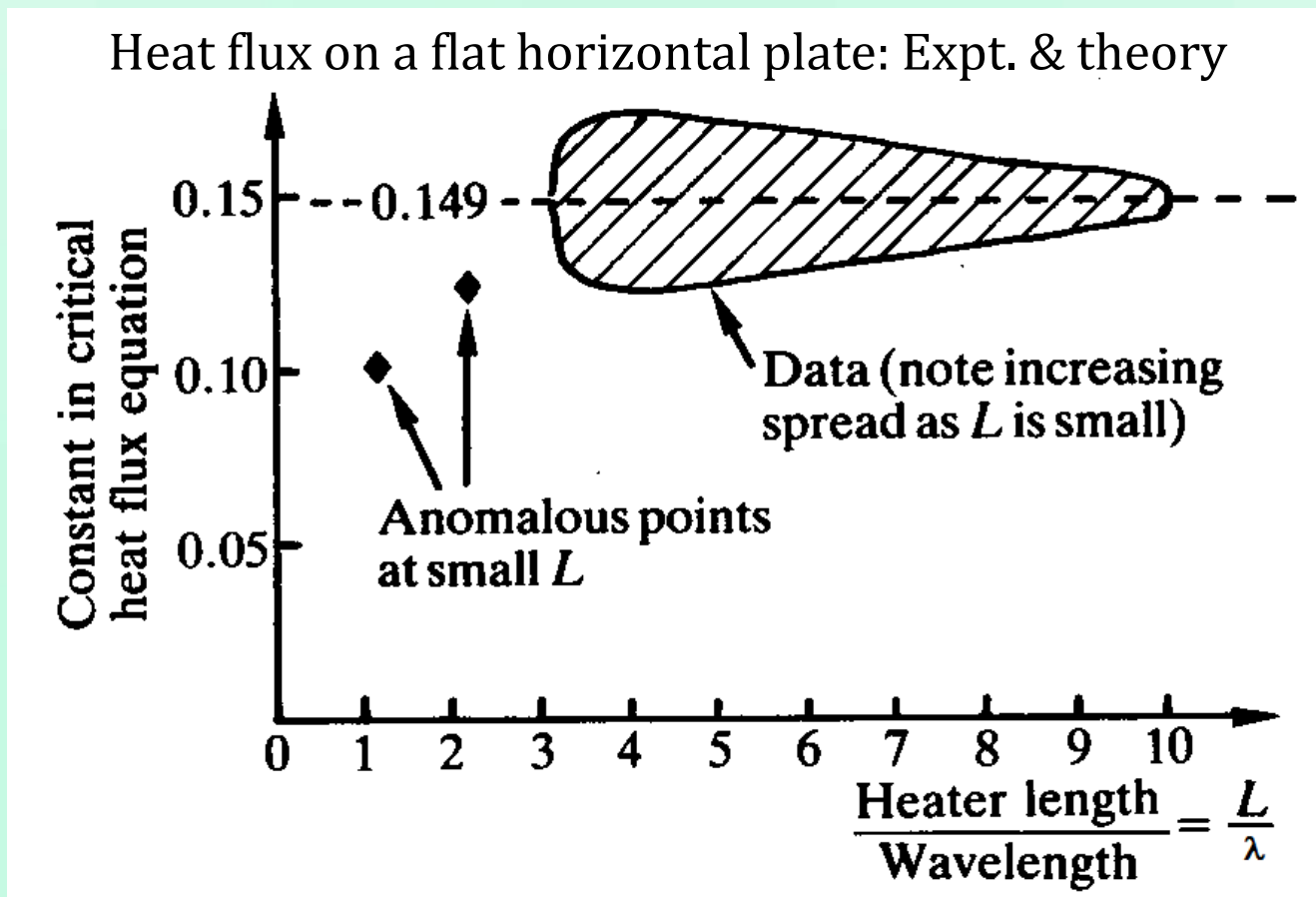
The coefficient modified for better fit for different geometries:

$$q''_{\max z} = 0.131 h_{lv} \rho_v^{1/2} [\sigma(\rho_l - \rho_v)g]^{1/4}$$

# Zuber's Correlation for Critical Heat Flux

$$q''_{\max} = C h_{lv} \rho_v^{1/2} [\sigma(\rho_l - \rho_v)g]^{1/4}$$

**$C = 0.149$  for Flat, Horizontal heater**

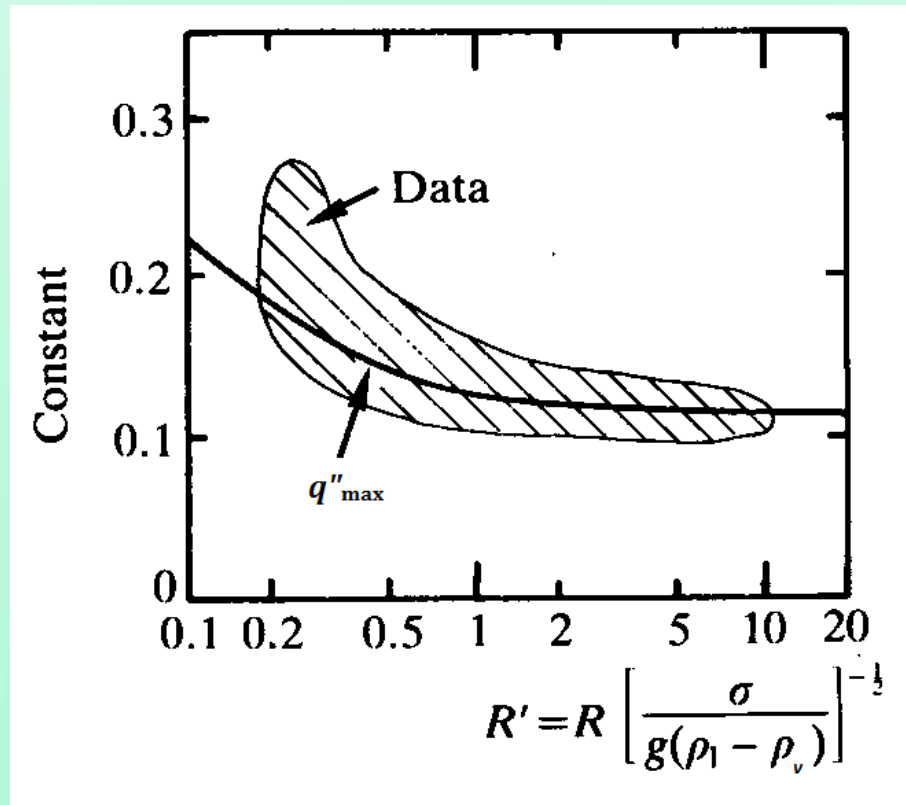


# Boiling outside Horizontal cylinders

$$q''_{\max} = C h_{lv} \rho_v^{1/2} [\sigma(\rho_l - \rho_v)g]^{1/4}$$

**$C = 0.116$  for Boiling outside Hor. Cyls**

Heat Flux outside Hor. Cyls: Expt. & theory

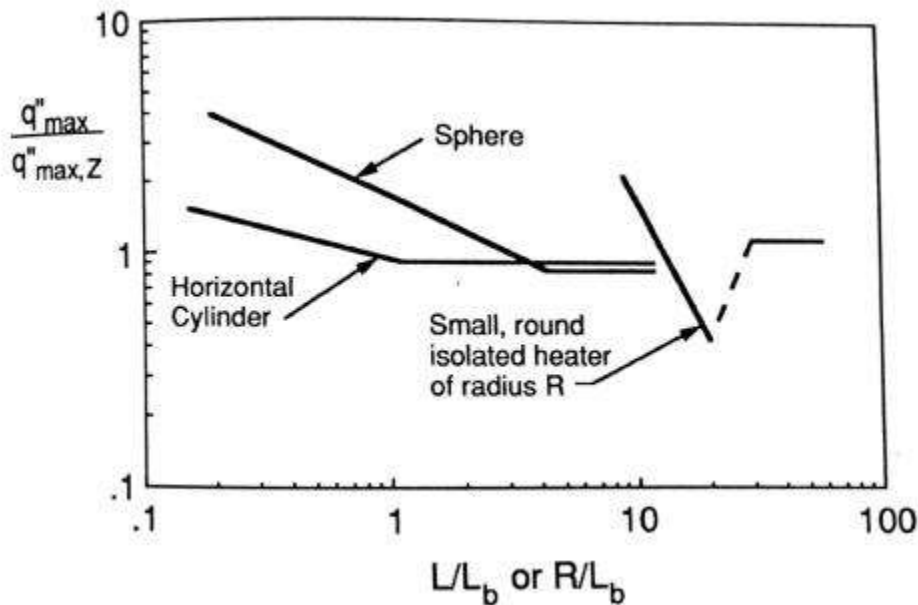


$$C = 0.116 + 0.3e^{-3.44R^{1/2}}$$

# Correlations for $q''_{\max}$ for different geometries of heaters

$$q''_{\max,z} = 0.131 h_{lv} \rho_v^{1/2} [\sigma (\rho_l - \rho_v) g]^{1/4}$$

$$L_b \sim \left[ \frac{\sigma}{g(\rho_l - \rho_v)} \right]^{1/2}$$



Geometry	Correlation	Range of applicability	R
Infinite heated flat plate	$\frac{q''_{\max}}{q''_{\max,Z}} = 1.14$	$\frac{L}{L_b} > 30$	
Small heater of width or diameter $L$ with vertical side walls	$\frac{q''_{\max}}{q''_{\max,Z}} = \frac{1.14 \lambda_D^2}{A_{\text{heater}}}$	$9 < \frac{L}{L_b} < 20$	
Horizontal cylinder of radius $R$	$\frac{q''_{\max}}{q''_{\max,Z}} = 0.89 + 2.27 \exp \left\{ -3.44 \sqrt{\frac{R}{L_b}} \right\}$	$\frac{R}{L_b} \leq 0.15$	
Large horizontal cylinder of radius $R$	$\frac{q''_{\max}}{q''_{\max,Z}} = 0.90$	$\frac{R}{L_b} > 1.2$	
Small horizontal cylinder of radius $R$	$\frac{q''_{\max}}{q''_{\max,Z}} = 0.94 \left( \frac{R}{L_b} \right)^{-1/4}$	$0.15 \leq \frac{R}{L_b} \leq 1.2$	
Large sphere of radius $R$	$\frac{q''_{\max}}{q''_{\max,Z}} = 0.84$	$4.26 \leq \frac{R}{L_b}$	
Small sphere of radius $R$	$\frac{q''_{\max}}{q''_{\max,Z}} = 1.734 \left( \frac{R}{L_b} \right)^{-1/2}$	$0.15 \leq \frac{R}{L_b} \leq 4.26$	
Small horizontal ribbon oriented vertically with side height $H$ – both sides heated	$\frac{q''_{\max}}{q''_{\max,Z}} = 1.18 \left( \frac{H}{L_b} \right)^{-1/4}$	$0.15 \leq \frac{H}{L_b} \leq 2.96$	
Small horizontal ribbon oriented vertically with side height $H$ – back side insulated	$\frac{q''_{\max}}{q''_{\max,Z}} = 1.4 \left( \frac{H}{L_b} \right)^{-1/4}$	$0.15 \leq \frac{H}{L_b} \leq 5.86$	
Small, slender, horizontal cylindrical body of arbitrary cross section with transverse perimeter $L_p$	$\frac{q''_{\max}}{q''_{\max,Z}} = 1.4 \left( \frac{L_p}{L_b} \right)^{-1/4}$	$0.15 \leq \frac{L_p}{L_b} \leq 5.86$	
Small bluff body with characteristic dimension $L$	$\frac{q''_{\max}}{q''_{\max,Z}} = C_0 \left( \frac{L}{L_b} \right)^{-1/2}$	Large $\frac{L}{L_b}$	

# Variation of Critical Heat Flux with Pressure

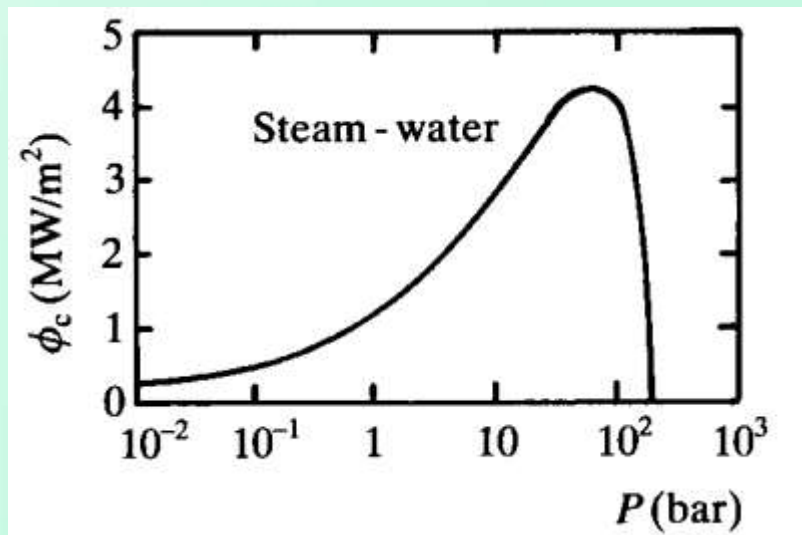
As the system pressure rises:

$h_{lv}$  falls slowly at first & falls steeply as the critical point is approached

$\rho_v$  increases monotonically

$\sigma$  falls monotonically, and

$\rho_l - \rho_v$  falls monotonically



$$q''_{\max} = 0.149 h_{lv} \rho_v^{1/2} [\sigma (\rho_l - \rho_v) g]^{1/4}$$

For both pool boiling and flow boiling the maximum critical heat flux occurs at about 70 bar.

Values of Critical Heat Flux for a flat, horizontal plate using steam-water

$P$ (bar)	$\phi_c$ (MW/m <sup>2</sup> )
0.01	0.168
0.1	0.471
1	1.25
10	2.97
30	4.03
50	4.38
70	4.45
90	4.34
100	4.10
150	3.27
221	0
$(p_c)$	

# Leidenfrost Temperature

## Rewetting of Hot Surfaces

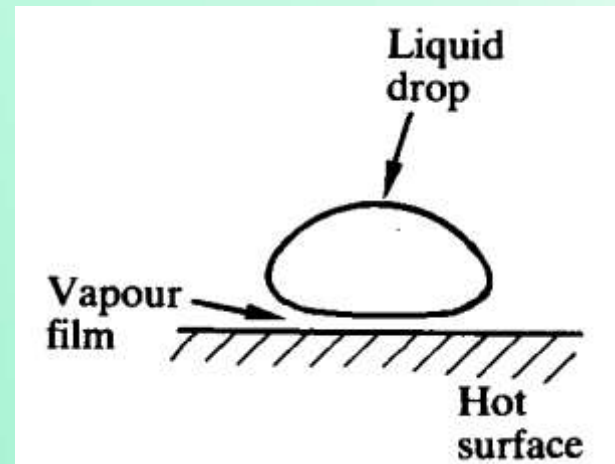
Liquid does not wet hot surface. Ex: A drop of water on a hot, horizontal plate will 'run' around in a chaotic manner and evaporate only slowly.

This occurs because the liquid is separated from the plate by a thin film of vapor so that the friction for sideways motion of the drop is very small and the heat transfer across the vapor film is poor.

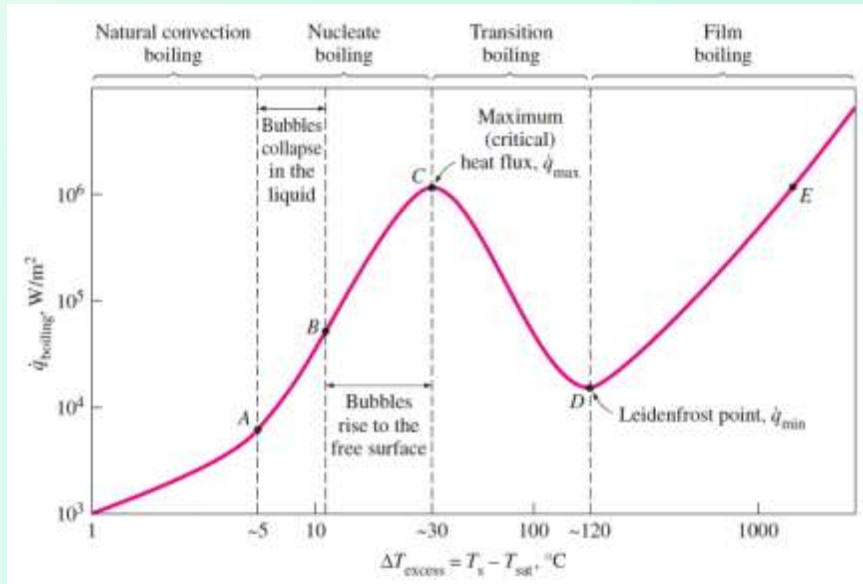
The vapor film, of course moves outwards, and fresh vapor is generated by evaporation at the underside of the drop due to heat conduction across the film and radiation from the plate to the drop.

If the plate is allowed to cool down, it will eventually reach a temperature at which the vapor film collapses, and then very intense boiling takes place which rapidly leads to the evaporation of all the liquid.

The surface temperature at which this sudden wetting of the plate occurs is the **Leidenfrost Temperature**.



# Leidenfrost: $q''_{\min}$



$$q''_{\min} = C h_{lv} \rho_v \left[ \frac{\sigma (\rho_l - \rho_v) g}{(\rho_l + \rho_v)^2} \right]^{1/4}$$

$C$  is a non-dimensional constant which lies between 0.09 and 0.18.

$C = 0.09$  provides a better fit.

$C = 0.13$  is sometimes taken as an intermediate value.

# Prof. Dr. Johann Gottlob Leidenfrost (1715-1794)

- Father – Minister
- Started off with Theological studies
- PhD thesis, “*On the Harmonious Relationship of Movements in the Human Body*”
- Professor at University of Duisburg
- Areas of influences
  - Theologian
  - Physician (Private Medical practice)
  - As a Prof. taught  
Medicine, Physics, and Chemistry



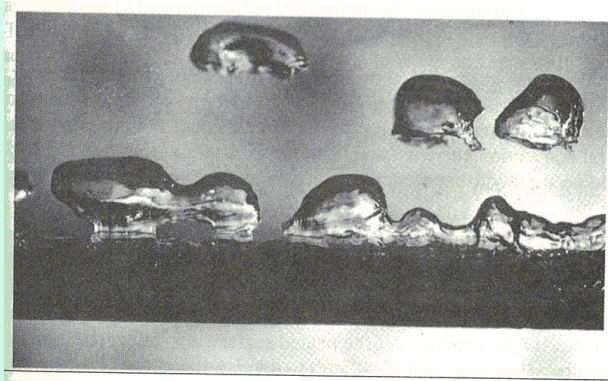
**70 Publications**

# **Multiphase Flow and Heat Transfer**

ME546

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# Film boiling for Large Horizontal Surfaces



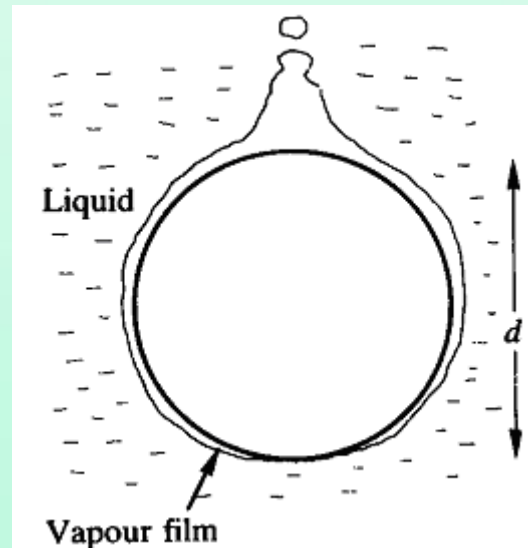
$$q''_{film\ boiling} = 0.425 \Delta T_{sat} \left[ \frac{\rho_v (\rho_l - \rho_v) g h'_{lv} k_v^3}{\mu_v \Delta T_{sat}} \left( \frac{(\rho_l - \rho_v) g}{\sigma} \right)^{1/2} \right]^{1/4}$$

$$h'_{lv} = h_{lv} + 0.5 C_{pv} \Delta T_{sat}$$

Effective latent heat of vaporization allowing for the inclusion of sensible heating effects in the vapor film.

To be precise, properties at  $T = T_w - \frac{\Delta T_{sat}}{2}$

# Film boiling for a Horizontal Cylinder of Diameter, $d$



$$q''_{film\ boiling} = 0.62 \Delta T_{sat} \left[ \frac{\rho_v (\rho_l - \rho_v) g h'_{lv} k_v^3}{\mu_v \Delta T_{sat} d} \right]^{1/4}$$

$$h'_{lv} = h_{lv} + 0.4 C_{pv} \Delta T_{sat}$$

To be precise, properties at  $T = T_w - \frac{\Delta T_{sat}}{2}$

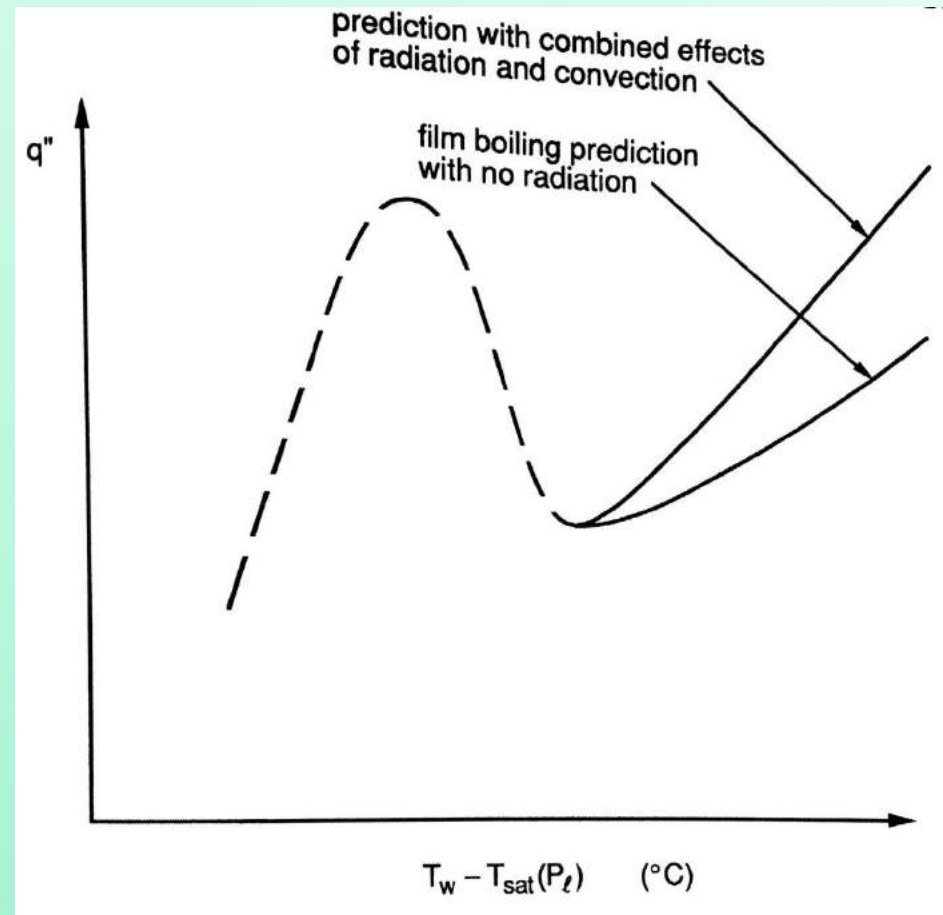
# Radiation Effects in Film boiling

$$h_{total} = h_{filmboiling} + \frac{3}{4} h_{radiative}$$

$$h_{radiative} = \sigma \varepsilon_{sur} \left( \frac{T_w^4 - T_{sat}^4}{T_w - T_{sat}} \right)$$

$\varepsilon_{sur}$  = surface emissivity

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$$



# Effect of Liquid Subcooling

**Subcooling:** In nucleate boiling as  $\Delta T_{sub} \uparrow$

$R_{max} \downarrow$  the bubble can grow

Bubble frequency,  $f \uparrow$

$q'' \propto R_{max}^3, f \text{ \& } n'$  (density of active nucleation sites)

A change  $\sim 300\%$  in  $\Delta T_{sub}$  produces  $\sim 20\%$  in  $\Delta q''$

**Superheating:** In nucleate boiling as  $\Delta T_{sat} \uparrow$

$R_{max}, f \text{ \& } n' \uparrow$

Critical radius,  $R \downarrow$  Number of active nucleation sites  $\uparrow$

# Effect of Liquid Subcooling

**Natural convection** portion will shift upward as driving  $\Delta T_{sub} \uparrow$

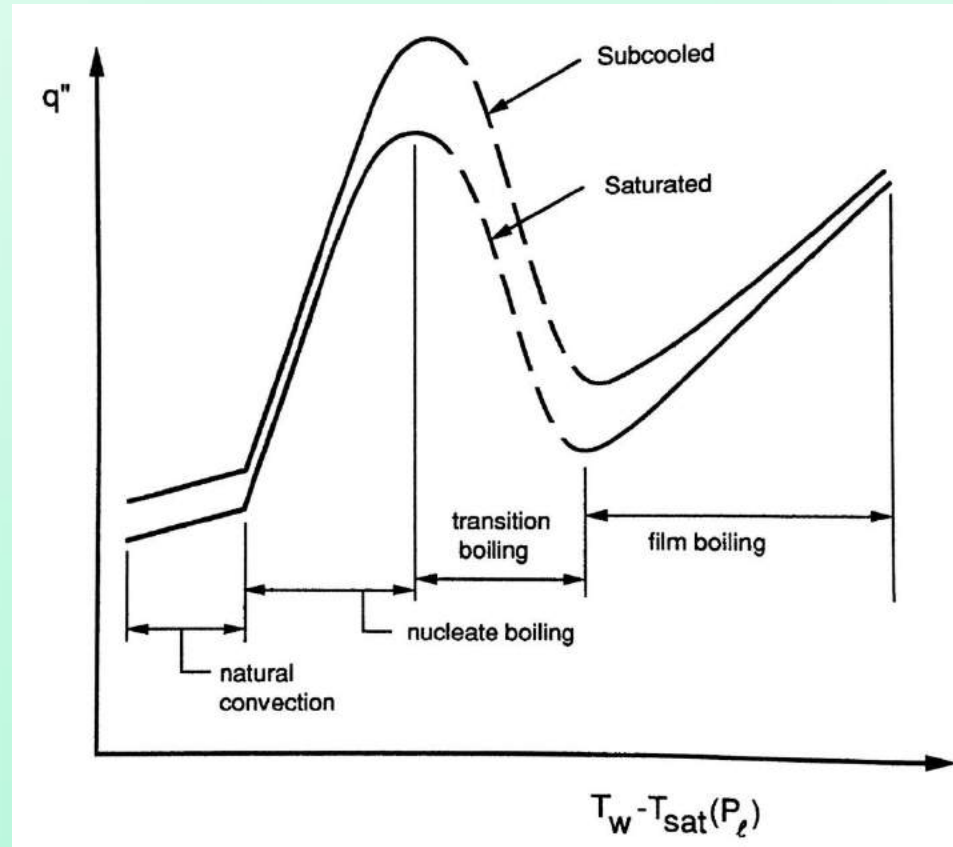
**Nucleate boiling:** slight influence as  $\Delta T_{sub} \uparrow$

A change  $\sim 300\%$  in  $\Delta T_{sub}$  produces  $\sim 20\%$  in  $\Delta q''$

**Maximum heat flux:** strong influence

Vapor raises and condenses – easy pathway for liquid to flow towards the surface

# Effect of Liquid Subcooling



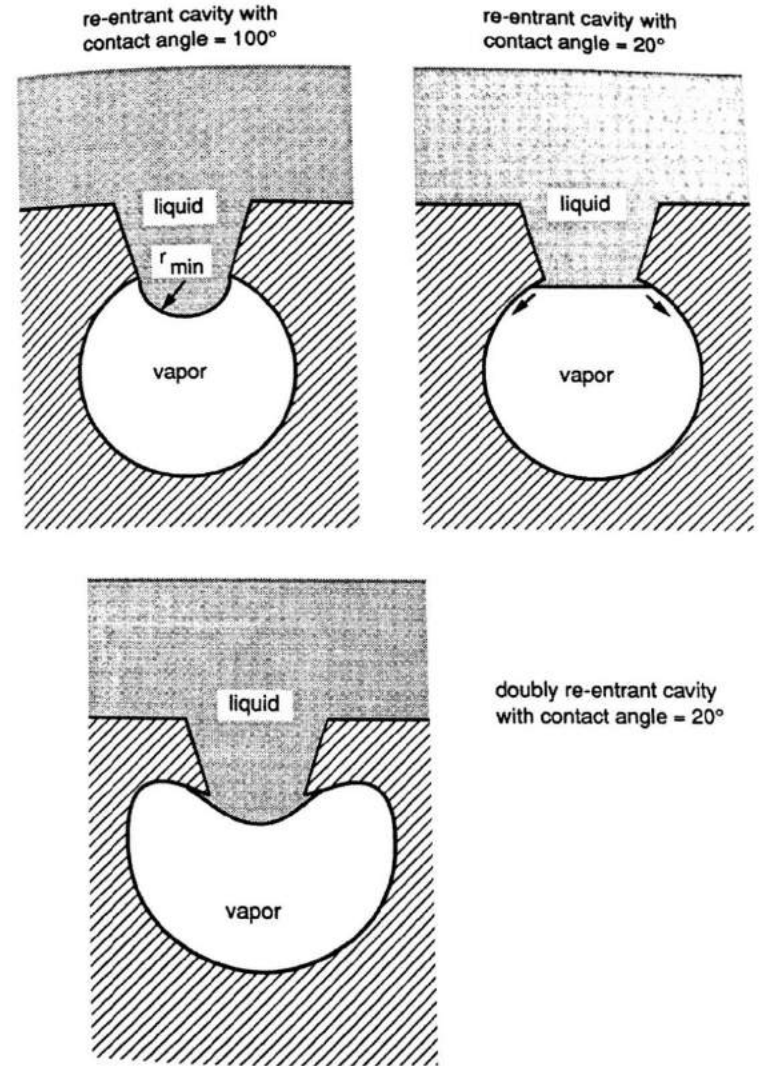
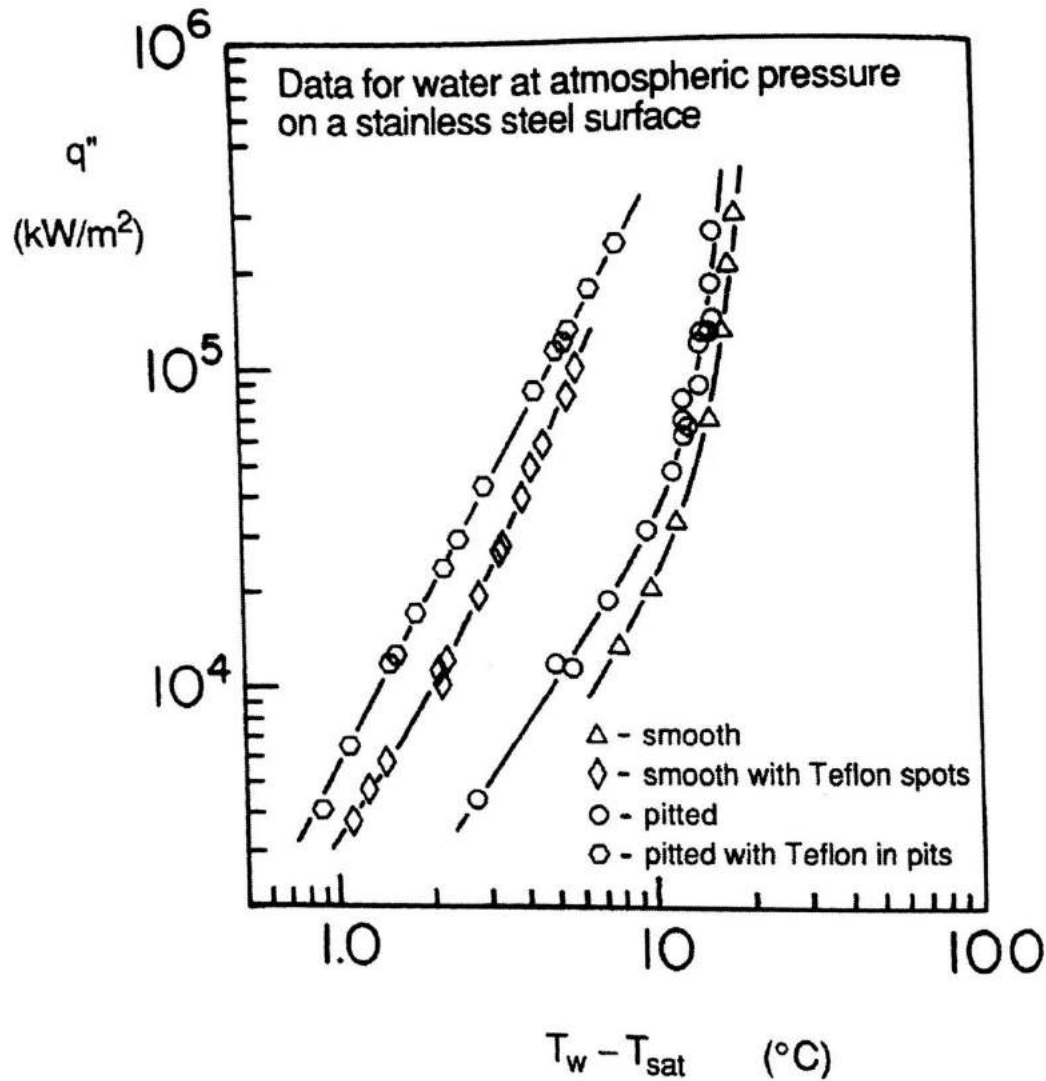
$$q''_{\max,sub} = 0.16h_{lv}\rho_v^{1/2}[\sigma(\rho_l - \rho_v)g]^{1/4} \left[ 1 + 0.1 \left( \frac{\rho_l}{\rho_v} \right)^{3/4} \frac{C_{pl} \Delta T_{sub}}{h_{lv}} \right]$$

# Enhancement of Heat Transfer During Boiling

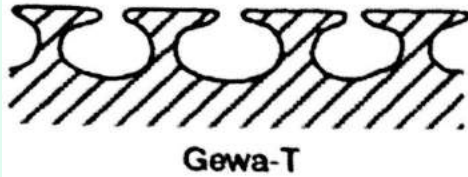
Augmentation of heat transfer during evaporation

- Roughening of the heating surface
- Structuring or coating the surface
- Production of artificial nucleation sites by sintering and
- Special shaping of the heating surface
- Addition of gases or liquids
- Addition of solids

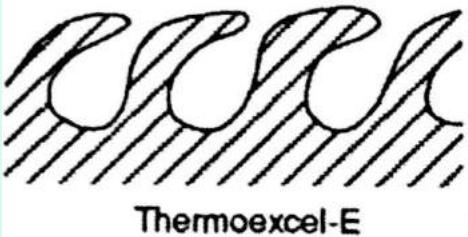
# Enhancement of Heat Transfer During Boiling



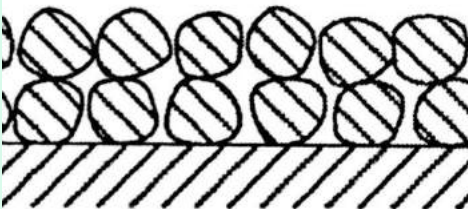
# Enhancement of Heat Transfer During Boiling



740 fins/m  
 Tube ID = 8 mm  
 Tube OD = 12.29 mm  
 1.1 mm fin height  
 0.25 mm gap

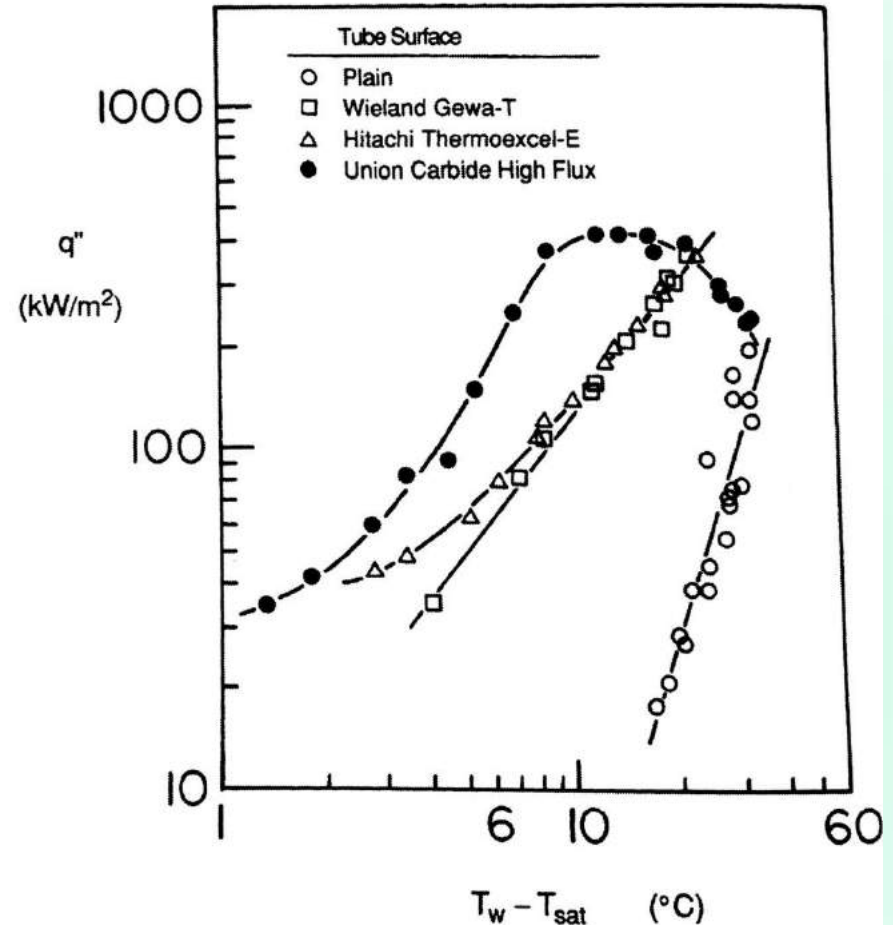


Tube ID = 10.61 mm  
 Tube OD = 13.16 mm  
 0.1 mm pore diameter  
 0.46 mm tunnel pitch  
 0.58 mm tunnel height



0.21 mm thick  
 46% < 44  $\mu\text{m}$   
 54% 44 to 74  $\mu\text{m}$   
 Tube OD = 13.31 mm

Pool boiling data for P-xylene at atmospheric pressure



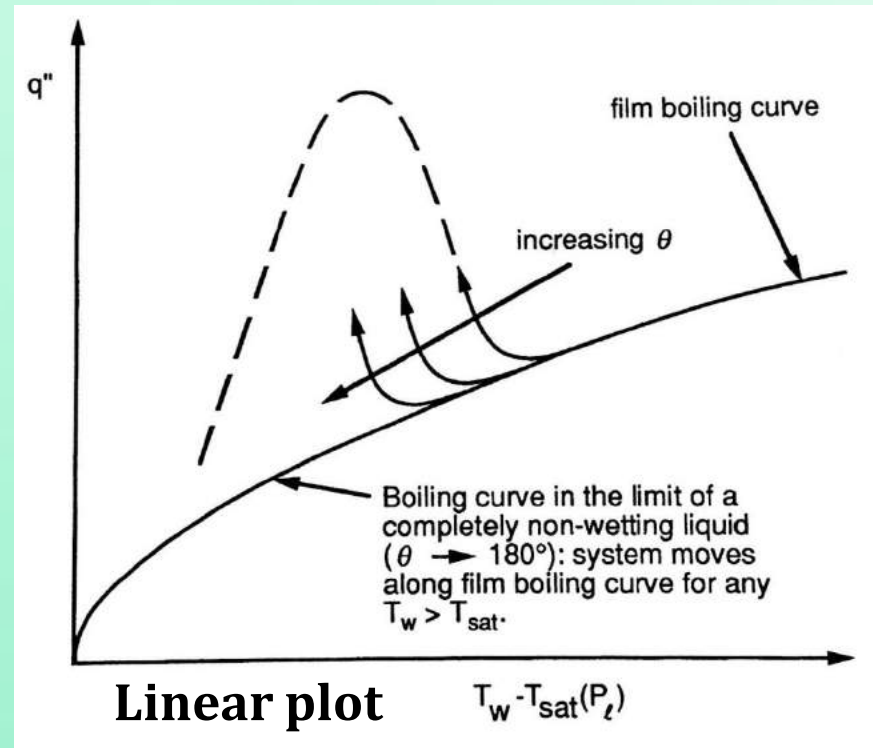
# Influence of Contact Angle

Boiling curve for a non-wetting liquid (Linear plot)

- $q'' \uparrow$  monotonically with superheat
- Eventually merges with the “classical” film boiling curve

Hydrophobic

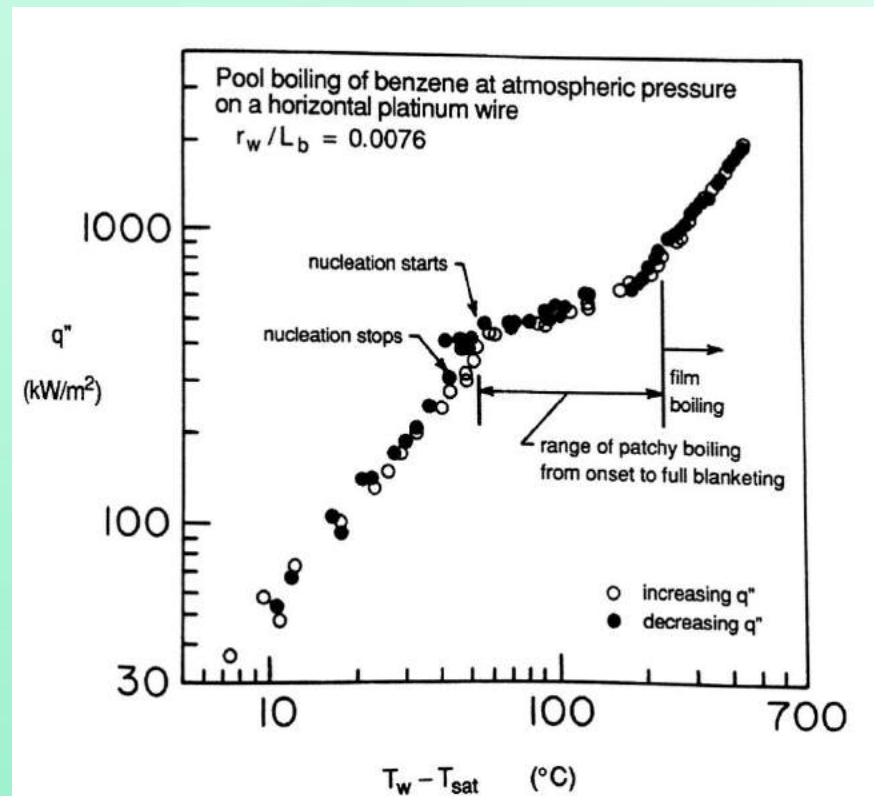
- Water – surface coated
- Mercury on Teflon



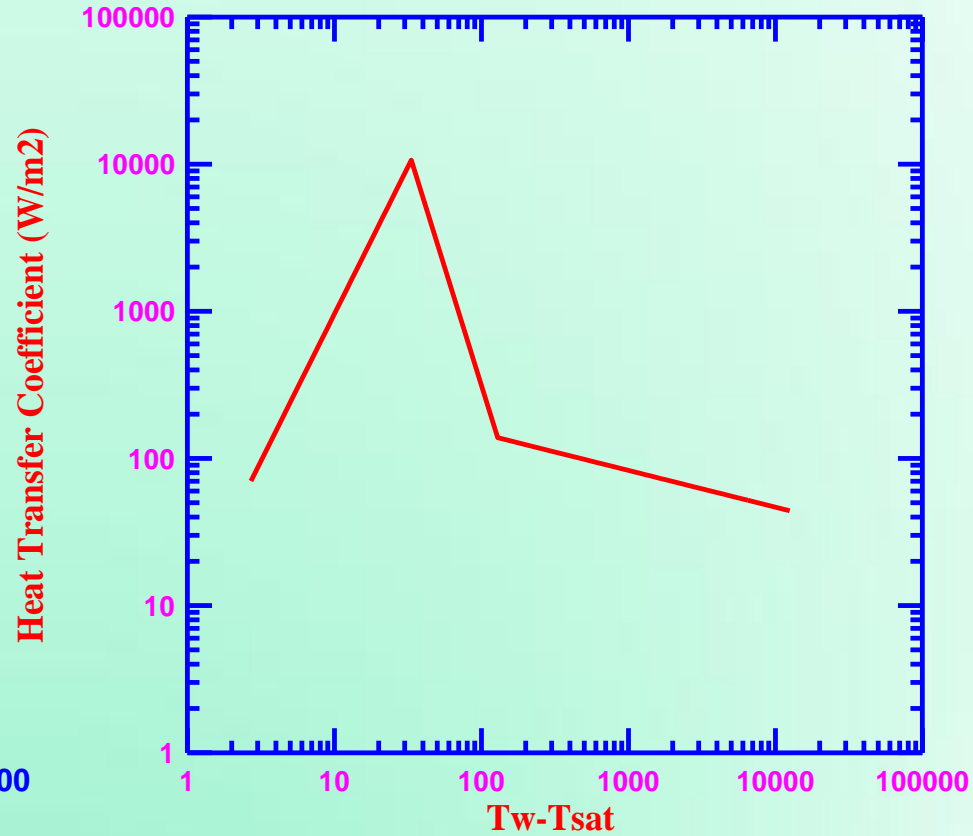
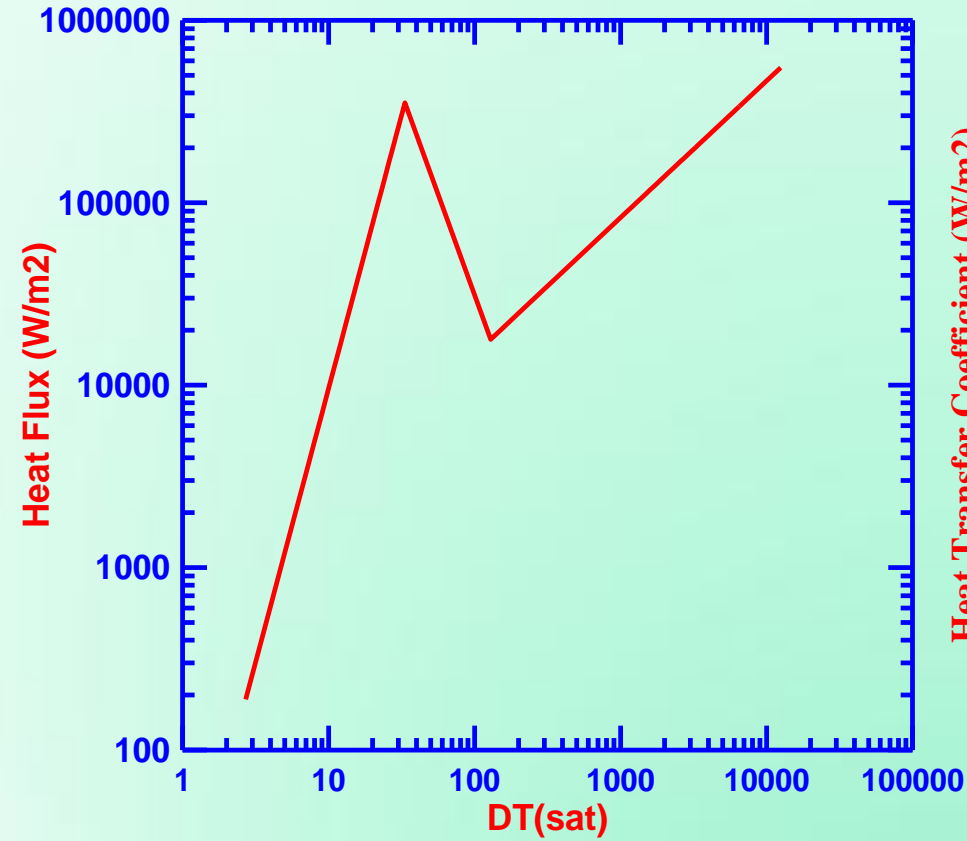
# Influence of Smaller Heater

Length scale,  $L_b \ll 50 D_b$  (Bubble departure diameter)

- Growing bubble completely covers the heater
- ONB initiates a film-type boiling
- No Nucleating regime
- No Transition regime
- No  $q''_{\max}$



# Pool Boiling curve of Benzene at 1 bar & $T_{sat} = 80^{\circ}\text{C}$



# Saturated Water-Steam on a Flat Horizontal Heater

$$T_w = 300^\circ\text{C}$$

Physical properties of water at 1 atm and 100°C:

$$T_{sat} = 100^\circ\text{C}$$

$$h_{lv} = 2257 \text{ kJ/kg}$$

$$k_v = 0.0251 \text{ W/m.K}$$

$$\mu_v = 12.3 \times 10^{-6} \text{ Pa.s}$$

$$\rho_v = 0.598 \text{ kg/m}^3$$

$$\rho_l = 958 \text{ kg/m}^3$$

$$\sigma = 0.0589 \text{ N/m}$$

$$C_{pv} = 2.029 \text{ kJ/kg.K}$$

$$q''_{\max} = 0.149 h_{lv} \rho_v^{1/2} [\sigma(\rho_l - \rho_v)g]^{1/4}$$

$$q''_{\min} = 0.09 h_{lv} \rho_v \left[ \frac{\sigma(\rho_l - \rho_v)g}{(\rho_l + \rho_v)^2} \right]^{1/4}$$

$$q''_{film} = 0.425 \Delta T_{sat} \left[ \frac{\rho_v (\rho_l - \rho_v) g h'_{lv} k_v^3}{\mu_v \Delta T_{sat}} \left( \frac{(\rho_l - \rho_v) g}{\sigma} \right)^{1/2} \right]^{1/4}$$

$$h'_{lv} = h_{lv} + 0.5 C_{pv} \Delta T_{sat}$$

# Saturated Water-Steam on a Flat Horizontal Heater

$$q''_{\max} = 1.26 \text{ MW/m}^2$$

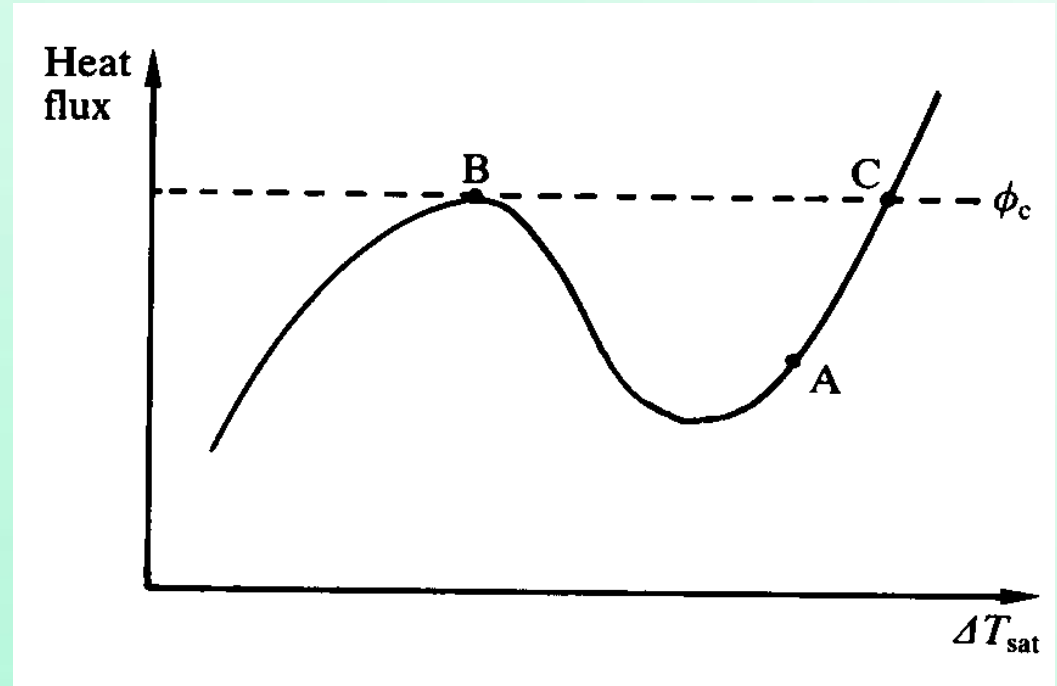
$$h_{\max} = 17.5 \text{ kW/m}^2\cdot\text{K}$$

$$q''_{\min} = 19.03 \text{ kW/m}^2$$

$$h_{\min} = 226 \text{ W/m}^2\cdot\text{K}$$

$$q''_{\text{film}} = 39.39 \text{ kW/m}^2$$

$$h_{\text{film}} = 185 \text{ W/m}^2\cdot\text{K}$$



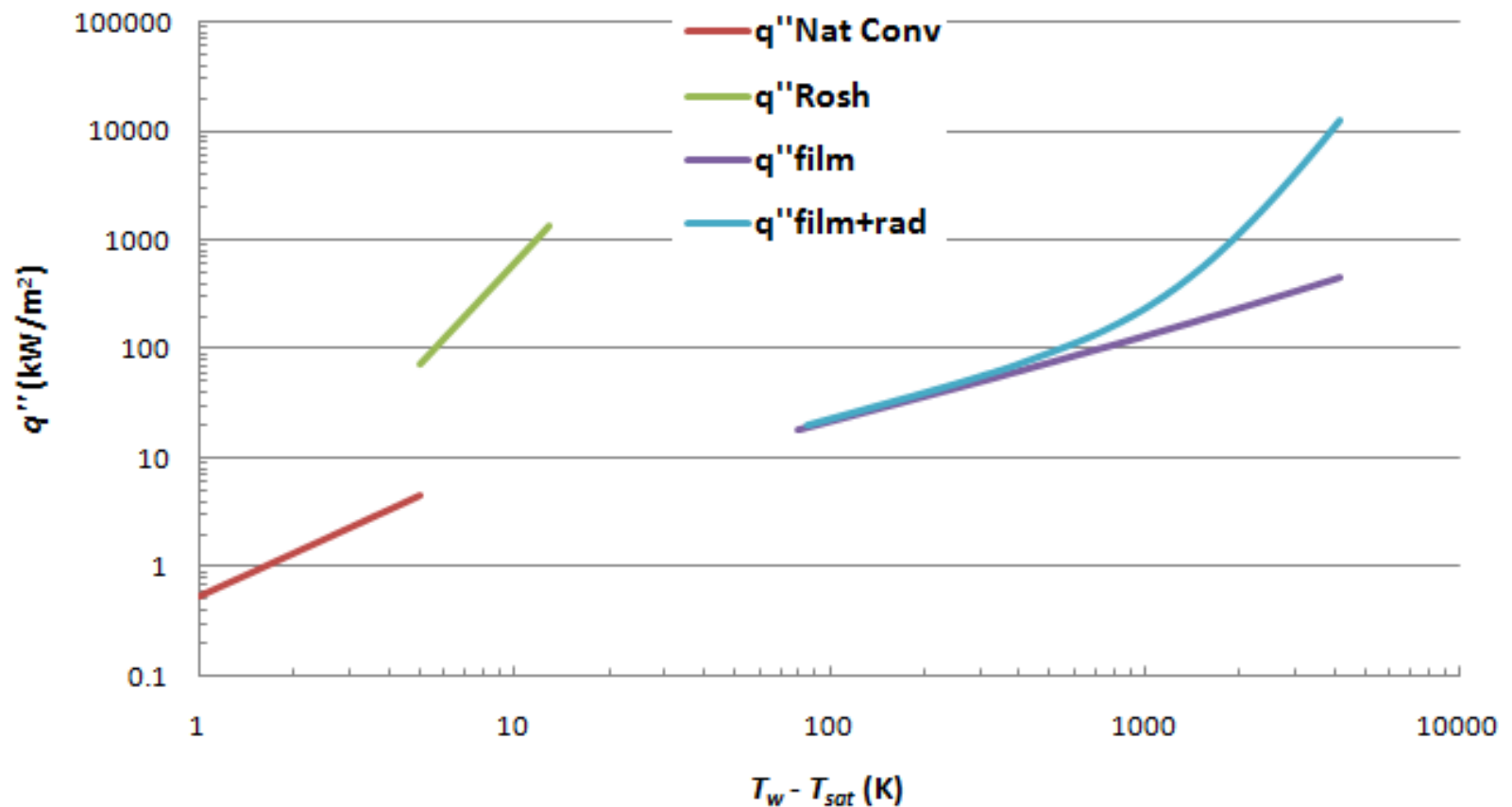
So, although the plate is very hot, it is carrying only a fraction of the critical heat flux. So we are around point A on the boiling curve.

The point C, which has the same heat flux as at point B, can be found.

$T_C$  is so high that radiative heat transfer is very important.

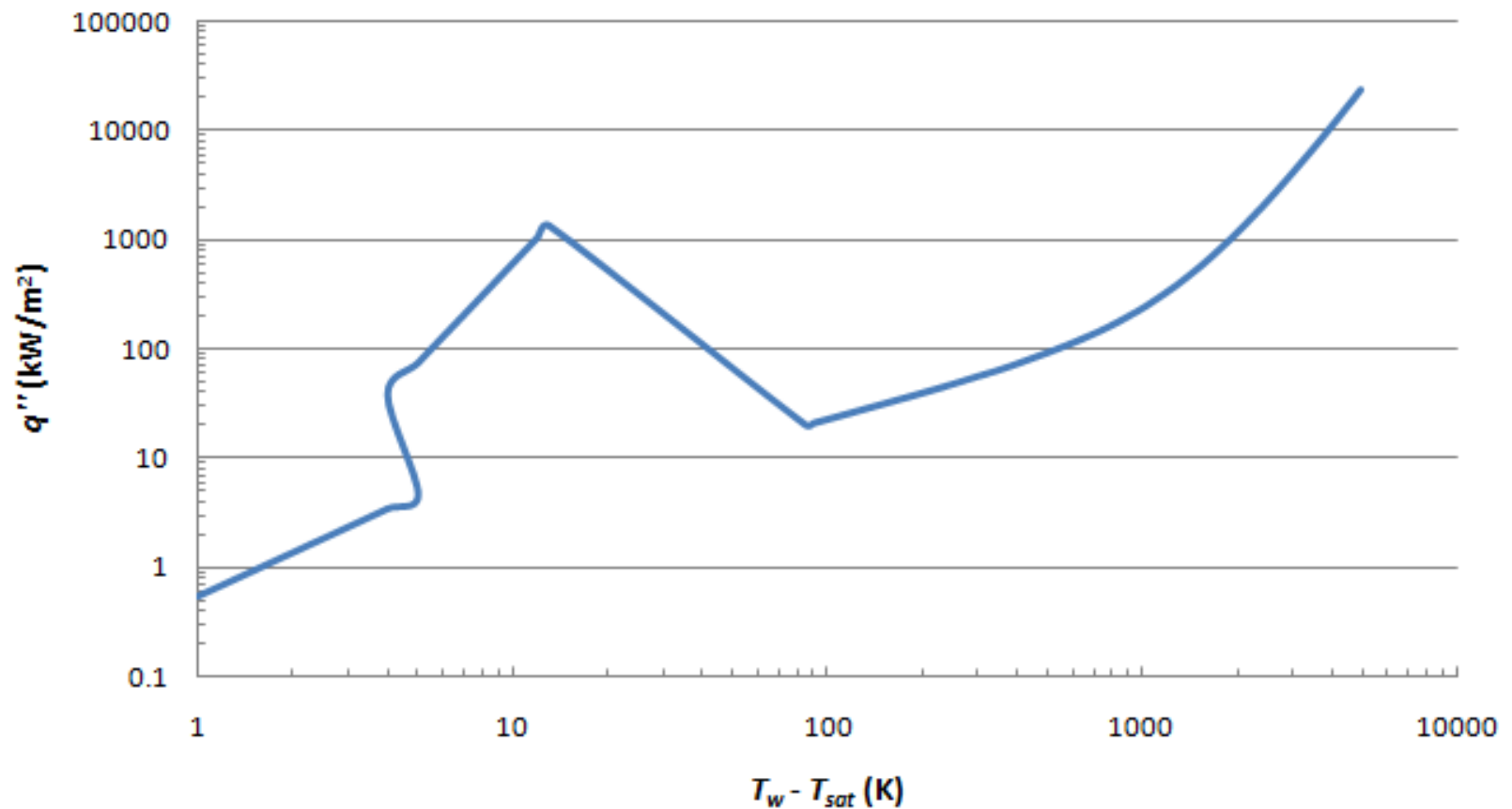
# Different Heat Transfer Scenarios

Boiling Curve Saturated Water-Steam at 1 atm on a flat horizontal ground & polished Stainless Steel



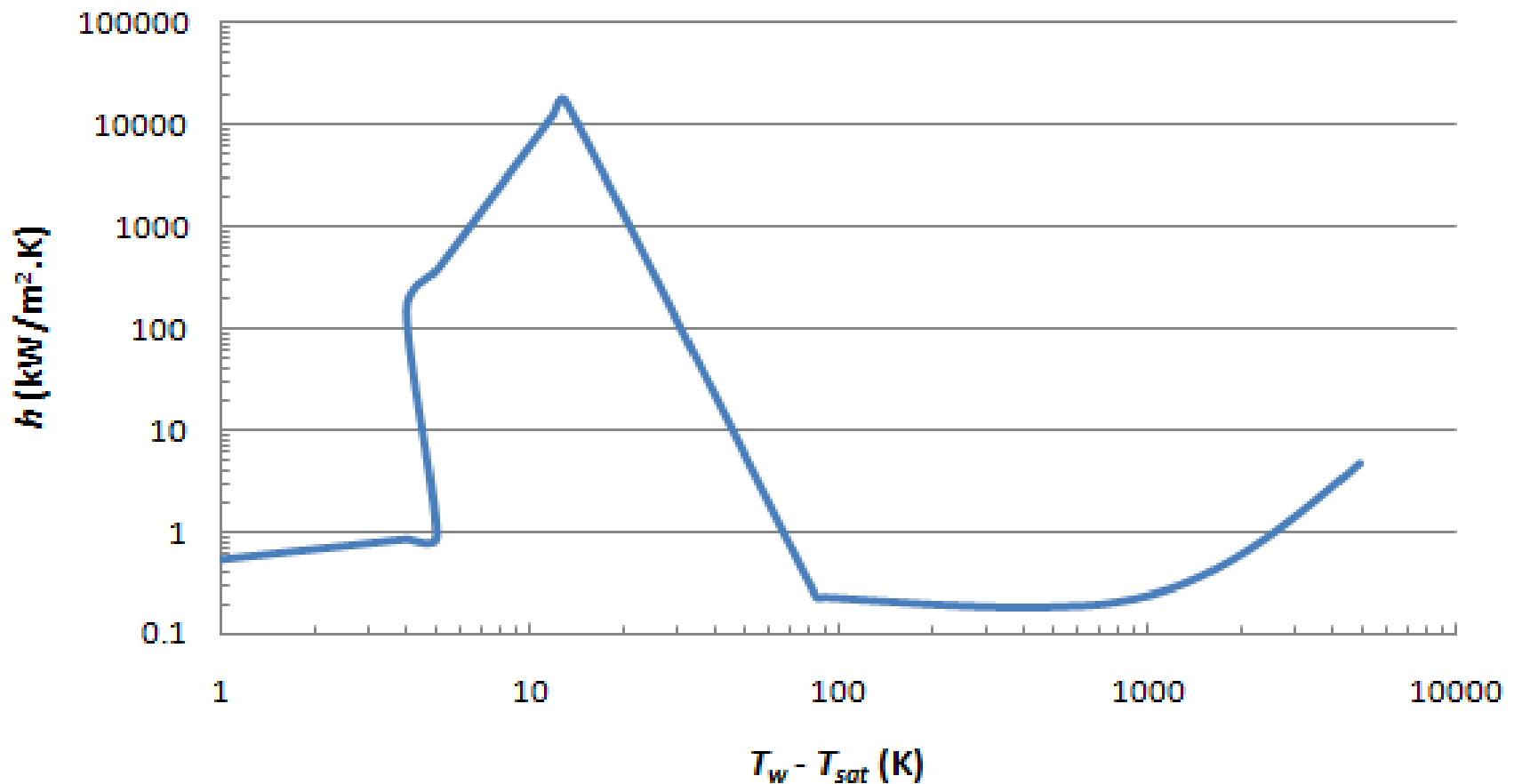
# Different Heat Transfer Scenarios

**Boiling Curve Saturated Water-Steam at 1 atm on a flat horizontal ground & polished Stainless Steel**



# Heat Transfer Coefficient in Different Scenarios

**Saturated Water-Steam at 1 atm on a flat horizontal ground & polished Stainless Steel**



# Problem on Maximum Heat Flux

Determine the critical heat flux for Trichlorotrifluoroethane (R113), nitrogen and water at atmospheric pressure for pool boiling from a horizontal cylinder with a diameter of 5 mm immersed in a pool of saturated liquid.

For saturated R-113 at atmospheric pressure:

$$T_{sat} = 320.7 \text{ K}, \rho_l = 1507 \text{ kg/m}^3, \rho_v = 7.46 \text{ kg/m}^3, h_{lv} = 146.3 \text{ kJ/kg}, \sigma = 0.0169 \text{ N/m}$$

For saturated nitrogen at atmospheric pressure:

$$T_{sat} = 77.4 \text{ K}, \rho_l = 807.1 \text{ kg/m}^3, \rho_v = 4.62 \text{ kg/m}^3, h_{lv} = 197.6 \text{ kJ/kg}, \sigma = 0.00885 \text{ N/m}$$

For saturated water at atmospheric pressure:

$$T_{sat} = 373 \text{ K}, \rho_l = 958.598 \text{ kg/m}^3, \rho_v = 0.598 \text{ kg/m}^3, h_{lv} = 2256 \text{ kJ/kg}, \sigma = 0.059 \text{ N/m}$$

# **Multiphase Flow and Heat Transfer**

ME546

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# Liquid-Vapor Interfacial Region (Nanoscale)

**In Macroscopic view**, the boundary between the bulk phases

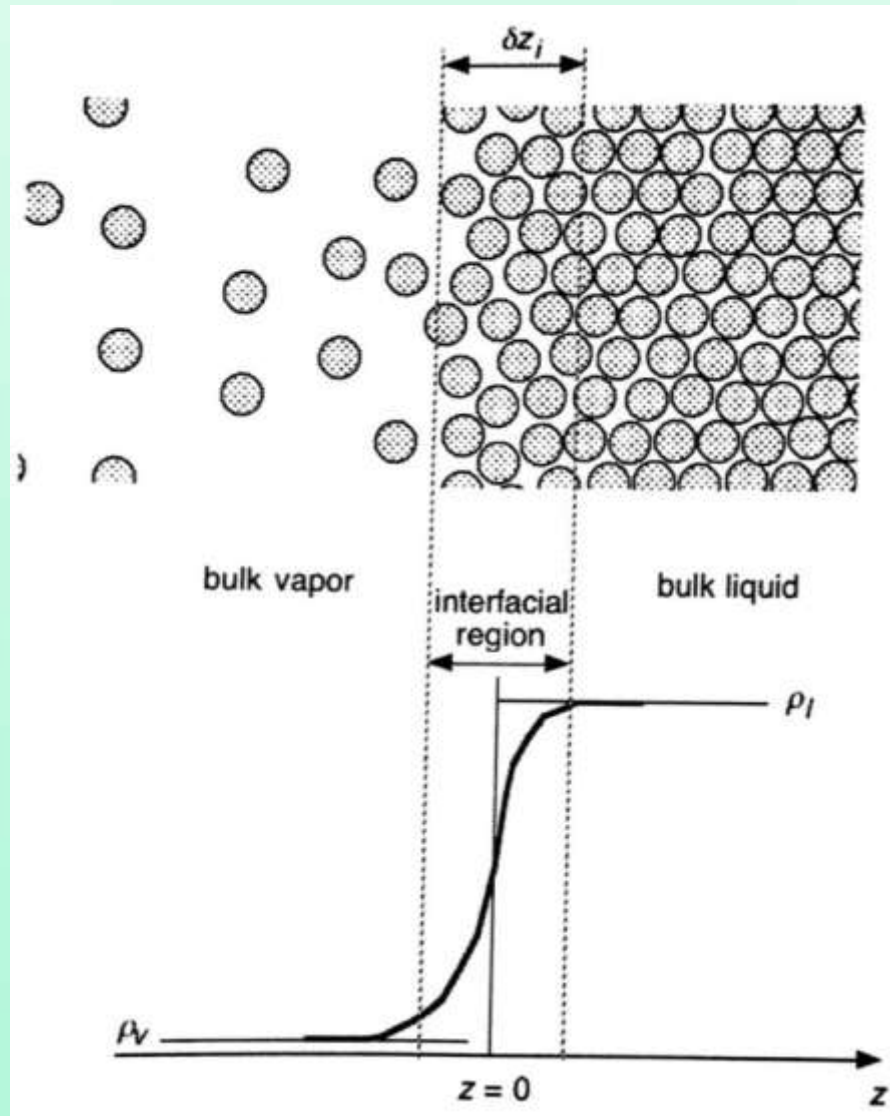
- Idealized as a surface
- Discontinuity in properties
- Net conversion of one phase into the other – Phase change

**In Nanoscale view**, the boundary between the bulk phases

- Actually a region
- A transition of mean molecular density exists
- This transition affects the thermophysics and transport in this region

Easy for analysis as a surface, but Nanoscale perspective gives better understanding.

# Liquid-Vapor Interfacial Region (Nanoscale)



Variation of the molecular density

# Interacting forces between two molecules

**At longer range**, two molecules exert attractive force.

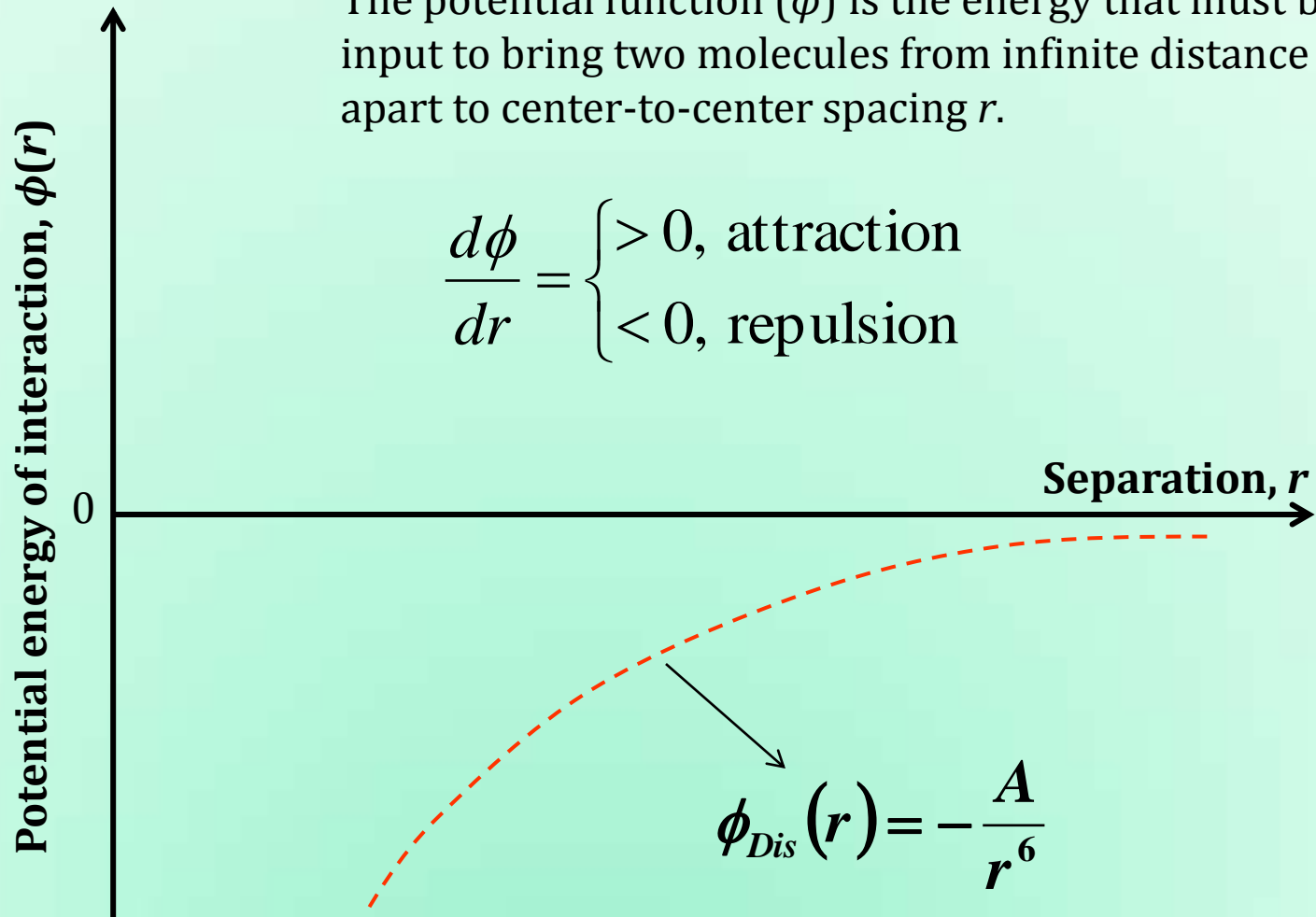
- Dipole-dipole forces or Electrostatic forces
  - Opposite sides of the molecule have opposite charges
- Dipole-induced forces
  - Permanently charged particle induces a dipole in a nearby neutral molecule
  - Strength depends on easy of polarization
- London interactions or Dispersion forces
  - Stronger interactions allow solid and liquid states to persist to higher temperatures.
  - Non-polar molecules show similar behavior, indicating that there are some types of intermolecular interactions that cannot be attributed to simple electrostatic attractions.

$$\phi_{Dis}(r) = -\frac{\lambda_{Dis}}{r^6}$$

# Interacting forces between two molecules

The potential function ( $\phi$ ) is the energy that must be input to bring two molecules from infinite distance apart to center-to-center spacing  $r$ .

$$\frac{d\phi}{dr} = \begin{cases} > 0, \text{ attraction} \\ < 0, \text{ repulsion} \end{cases}$$



$A$  is a constant varies with the type of molecule, polarizability of the molecules.

# Liquid-Vapor Interfacial Region (Nanoscale)

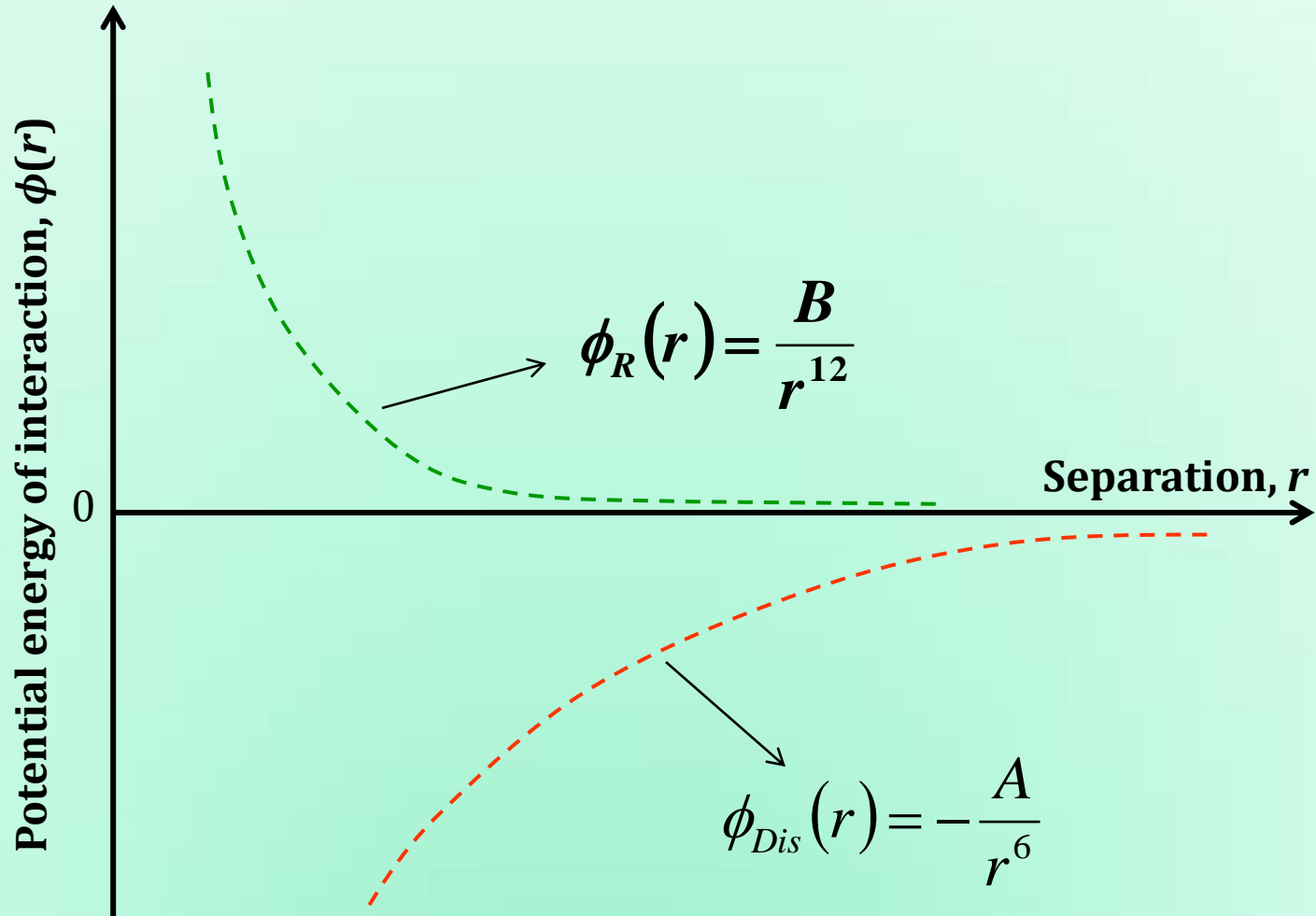
**At very short range**, two molecules exert a repulsive force

- Interference of the electron orbits of one molecule with those of the other
- The energy required to bring two molecules from infinite distance apart to center-to-center spacing,  $r$

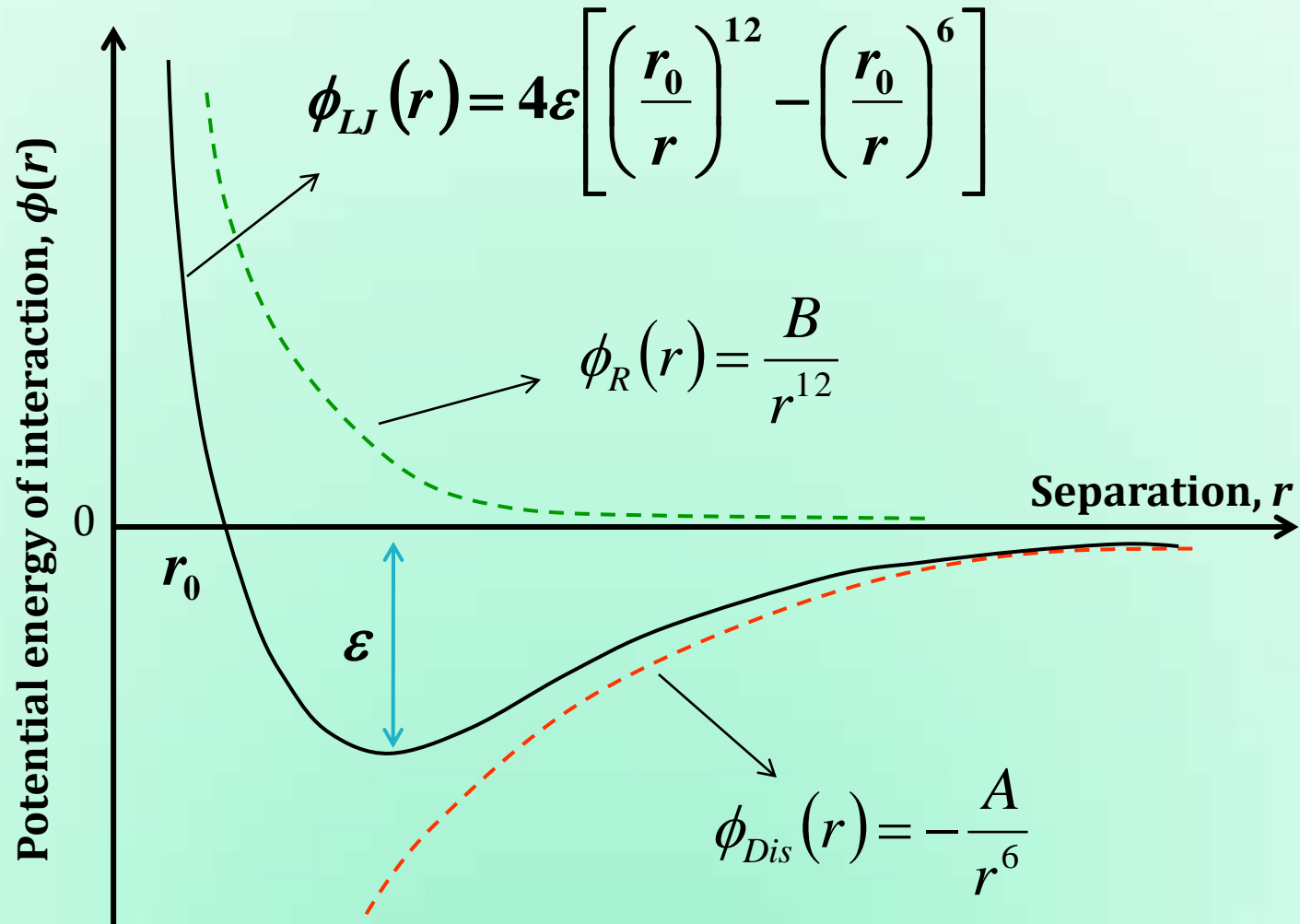
$$\phi_R(r) = \frac{B}{r^k}, 9 \leq k \leq 15$$

where  $B$  is a constant depending on the type of molecule.

# Interacting forces between two molecules



# Lennard-Jones 6-12 potential



# Interacting forces between two molecules

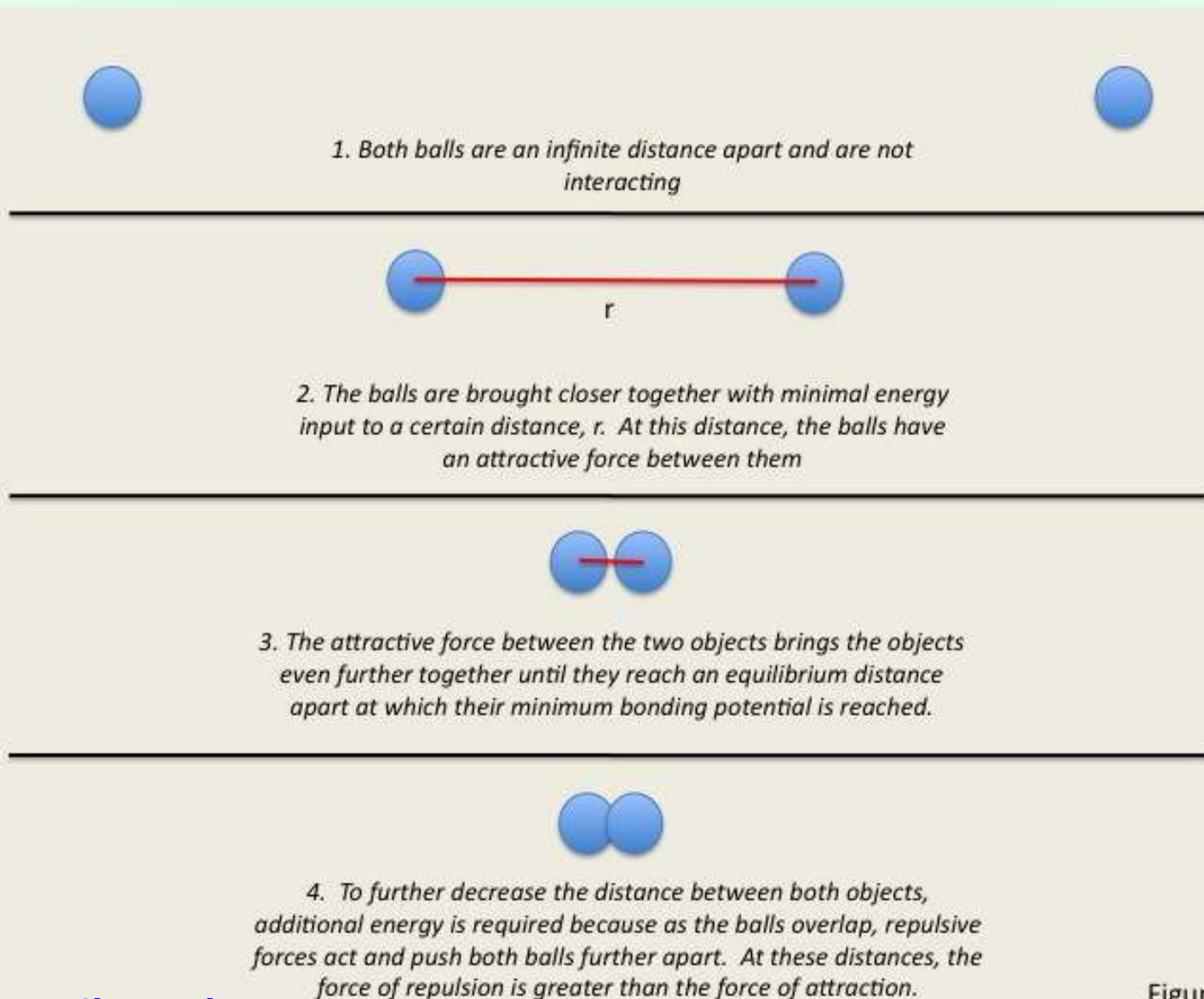
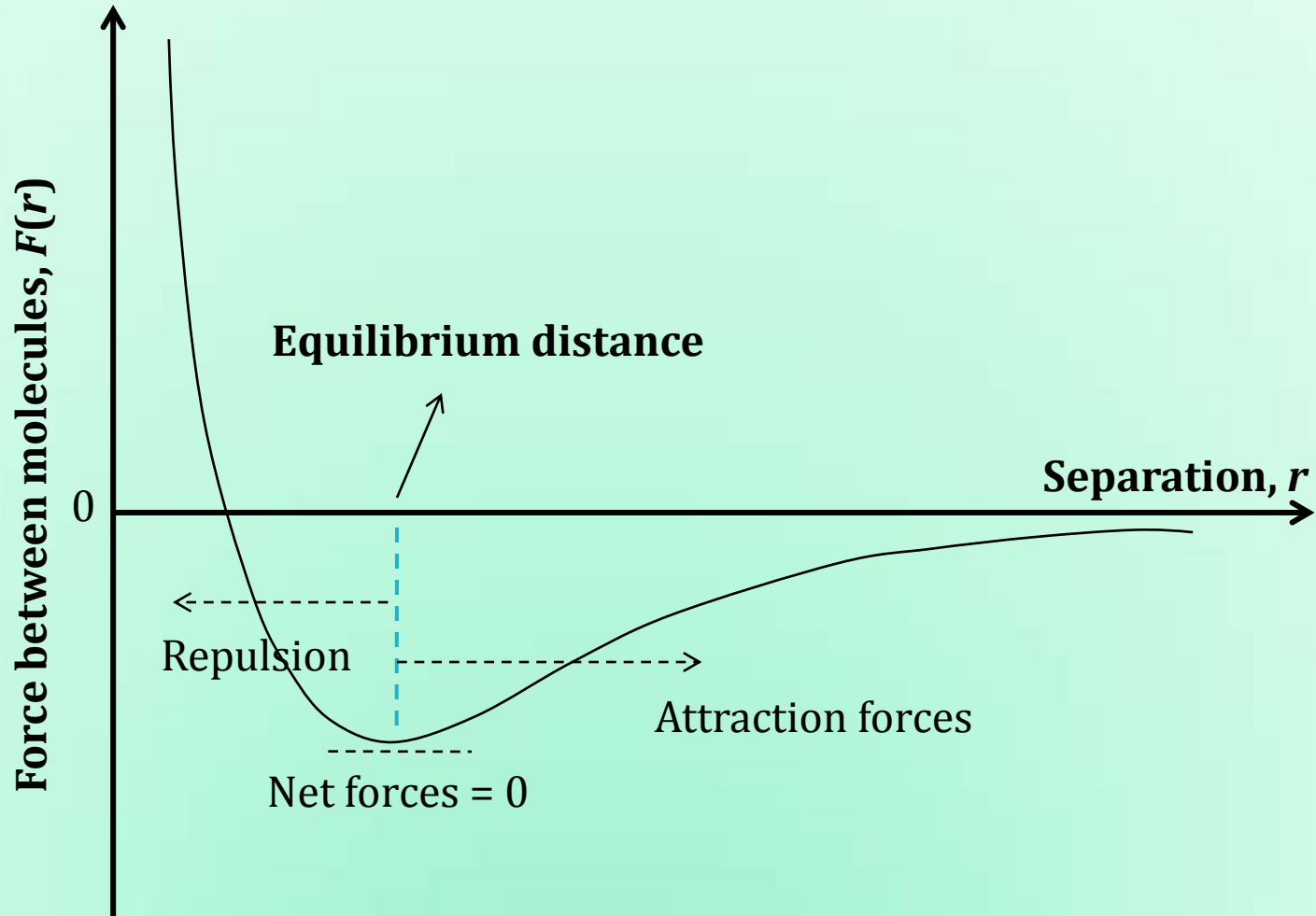


Figure A

# Lennard-Jones 6-12 potential



# Lennard-Jones 6-12 potential

$$\phi_{LJ}(r) = 4\epsilon \left[ \left( \frac{r_0}{r} \right)^{12} - \left( \frac{r_0}{r} \right)^6 \right]$$

- To bring two molecules that are initially very far apart into closer proximity, we must remove energy - **Condensation**
- If two molecules are close enough to feel attractive forces, but not so close that repulsive forces come into play, then energy must be supplied to increase the spacing of the molecules - **Vaporization**

# Lennard-Jones 6-12 potential

$$\phi_{LJ}(r) = 4\epsilon \left[ \left( \frac{r_0}{r} \right)^{12} - \left( \frac{r_0}{r} \right)^6 \right]$$

$\epsilon$  is the depth of potential well (J)

- Energy that must be input for one molecule to escape the attractive pull of another.
- In a two-phase system, escape of molecules from a liquid phase into a vapor phase at the interface is more probable if the translational kinetic energy is larger than  $\epsilon$ .

$r_0$  is the distance at which potential energy is zero (nm).

# Lennard-Jones 6-12 potential

$$\phi_{LJ}(r) = 4\epsilon \left[ \left( \frac{r_0}{r} \right)^{12} - \left( \frac{r_0}{r} \right)^6 \right]$$

$\epsilon$  is the depth of potential well (J)       $\phi_{LJ} = -\epsilon \Big|_{r=2^{1/6}r_0}$

- Energy that must be input for one molecule to escape the attractive pull of another.
- In a two-phase system, escape of molecules from a liquid phase into a vapor phase at the interface is more probable if the translational kinetic energy is larger than  $\epsilon$ .

$r_0$  is the distance at which potential energy is zero (nm).

# Maxwell-Boltzmann Distribution

Number of molecules with speed in the interval  $c$  to  $c + dc$ :

$$dN_c = 4\pi N \left( \frac{m}{2\pi k_B T} \right)^{\frac{3}{2}} c^2 e^{\frac{-mc^2}{2k_B T}} dc$$

$N$  is the number of molecules

$c$  is the speed of the molecule, m/s

$m$  is the mass of the molecule, kg

$k_B$  is the Boltzmann constant,  $1.38 \times 10^{-23} \text{ m}^2\text{kg/s}^2\cdot\text{K}$

$T$  is the absolute temperature, K

# Maxwell-Boltzmann Distribution

We are interested in knowing the number of molecules that would have kinetic energy more than the minimum potential (depth of the potential well).

Convert speed distribution to kinetic energy of a molecule.

$$K = \frac{1}{2}mc^2$$

# Maxwell-Boltzmann Distribution

We are interested in knowing the number of molecules that would have kinetic energy more than the minimum potential (depth of the potential well).

Convert speed distribution to kinetic energy of a molecule.

$$K = \frac{1}{2} m c^2$$

$$c = \sqrt{\frac{2K}{m}} \quad dc = \sqrt{\frac{1}{2mK}} dK$$

$$dN_K = 2\pi N (\pi k_B T)^{-\frac{3}{2}} K^{\frac{1}{2}} e^{\frac{-K}{k_B T}} dK$$

# Maxwell-Boltzmann Distribution

The number of molecules in the gas with energies above  $\varepsilon$ ,  $N_{>\varepsilon}$

$$N_{>\varepsilon} = \int_{\varepsilon}^{\infty} dN_K$$

The fraction of molecules with energies above  $\varepsilon$  is  $N_{>\varepsilon}/N$

$$\frac{N_{>\varepsilon}}{N} = \left( \frac{4\varepsilon}{\pi k_B T} \right)^{\frac{1}{2}} e^{-\frac{\varepsilon}{k_B T}} + \operatorname{erfc} \left( \sqrt{\frac{-\varepsilon}{k_B T}} \right)$$

Error function term can be neglected if  $\varepsilon \gg k_B T$

# Equilibrium Vapor Pressure

- Molecules having energies above the threshold value ( $\epsilon$ ) increase rapidly with temperature.
- This determines chemical reaction rates and the equilibrium conditions in two-phase systems.
- Even at low temperatures some fraction of the molecules in the liquid will have sufficient energy to escape the cohesive forces of other liquid molecules at the liquid-vapor interface.
- The fraction capable of escaping in this manner will increase rapidly with temperature.

# Vapor Pressure & Latent Heat of Vaporization

- A liquid with a small cohesive energy will have a higher vapor pressure than one with a large cohesive energy.
- In general, cohesive energy  $\propto \varepsilon$  (Lennard-Jones potential).
- $H_{lv} \cong$  cohesive energy of the liquid
- At same temperature, a liquid with a high  $H_{lv}$  should have a lower vapor pressure than a liquid with a smaller  $H_{lv}$ .
- At 20°C: Water

$$P_{sat} = 2.34 \text{ kPa and } H_{lv} = 2454 \text{ kJ/kg}$$

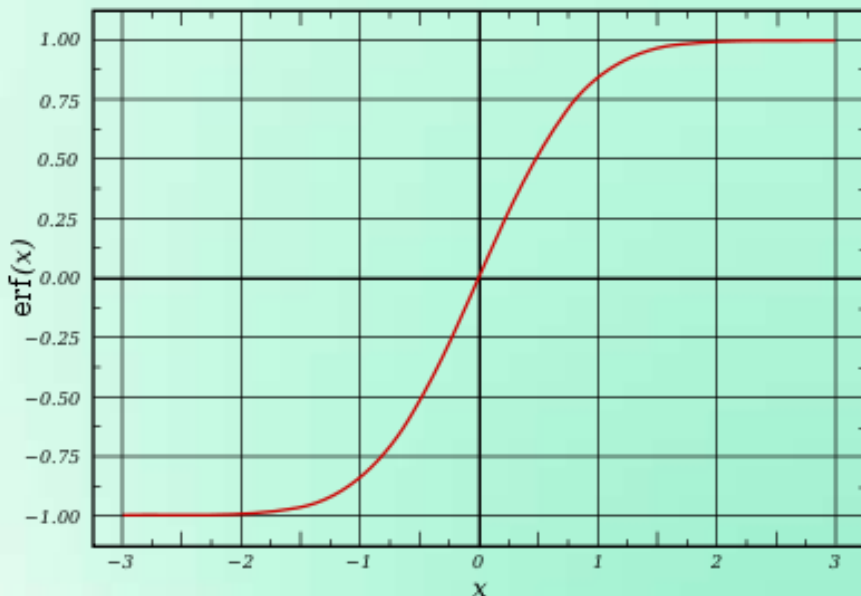
- Saturated refrigerant-134a

$$P_{sat} = 572.1 \text{ kPa and } H_{lv} = 182.3 \text{ kJ/kg}$$

# Latent Heat of Vaporization

For saturated nitrogen at 77 K, estimate the fraction of the molecules that have translational kinetic energies larger than,  $\varepsilon = 1.31 \times 10^{-21}$  J.

$$\frac{N_{>\varepsilon}}{N} = \left( \frac{4\varepsilon}{\pi k_B T} \right)^{\frac{1}{2}} e^{-\frac{\varepsilon}{k_B T}} + \operatorname{erfc} \left( \sqrt{\frac{-\varepsilon}{k_B T}} \right)$$



$$k_B = 1.38 \times 10^{-23} \text{ m}^2\text{kg/s}^2\cdot\text{K}$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

# Latent Heat of Vaporization

For saturated nitrogen at 77 K, estimate the fraction of the molecules that have translational kinetic energies larger than,  $\varepsilon = 1.31 \times 10^{-21}$  J.

$$\frac{N_{>\varepsilon}}{N} = \left( \frac{4\varepsilon}{\pi k_B T} \right)^{\frac{1}{2}} e^{-\frac{\varepsilon}{k_B T}} + \operatorname{erfc} \left( \sqrt{\frac{-\varepsilon}{k_B T}} \right)$$

$$\frac{\varepsilon}{k_B T} = 1.23$$

$$\frac{N_{>\varepsilon}}{N} = 0.483$$

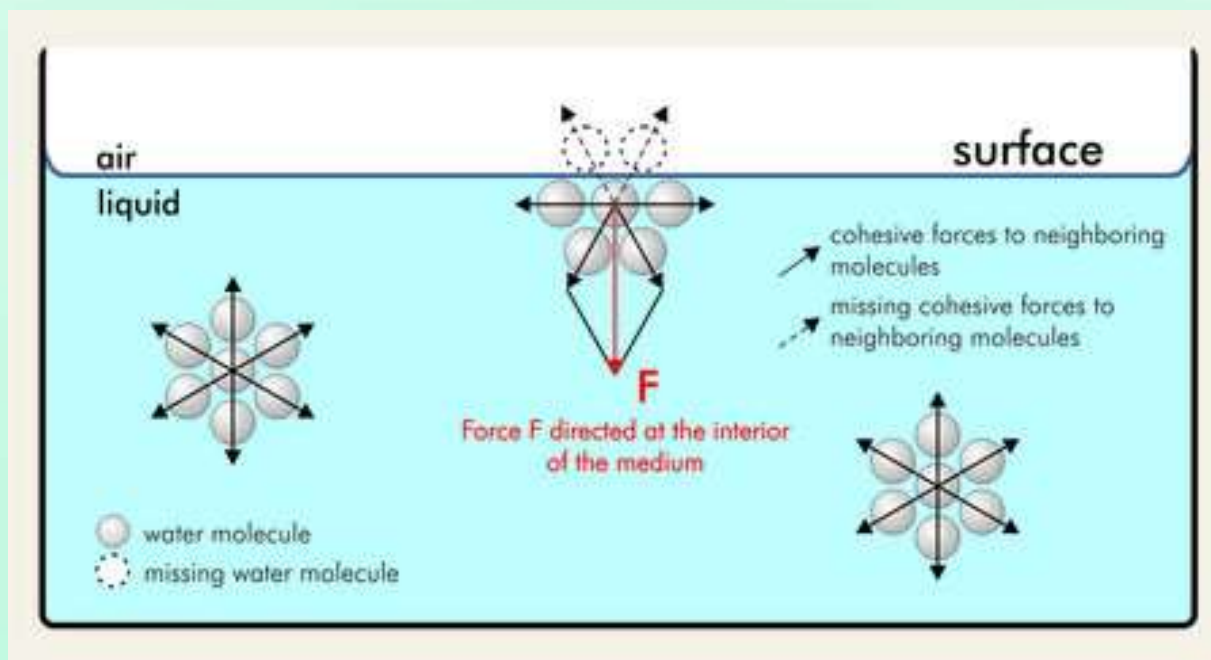
# **Multiphase Flow and Heat Transfer**

ME546

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# Surface Tension

The net attractive force causes the liquid surface to contract inwards until repulsion from other surrounding molecules start dominating.



If there are no external forces, spherical interface is formed.

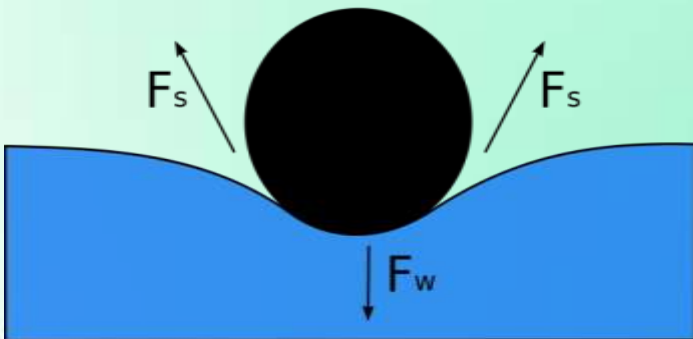
Intermolecular attraction – Interfacial Tension.

# Surface Tension

Practically, the phenomenon is comparable to a thin layer surrounding the liquid and making it hard to dip an object in and to pull it out of the liquid.

Against force ( $F$ ) work has to be performed to move something through this layer or to expand its surface.

Hence,  $\sigma$  is the amount of force (Nm) necessary to expand the surface ( $m^2$ ) of a liquid by one unit.



# Van der Waals (molecular) theory of capillarity

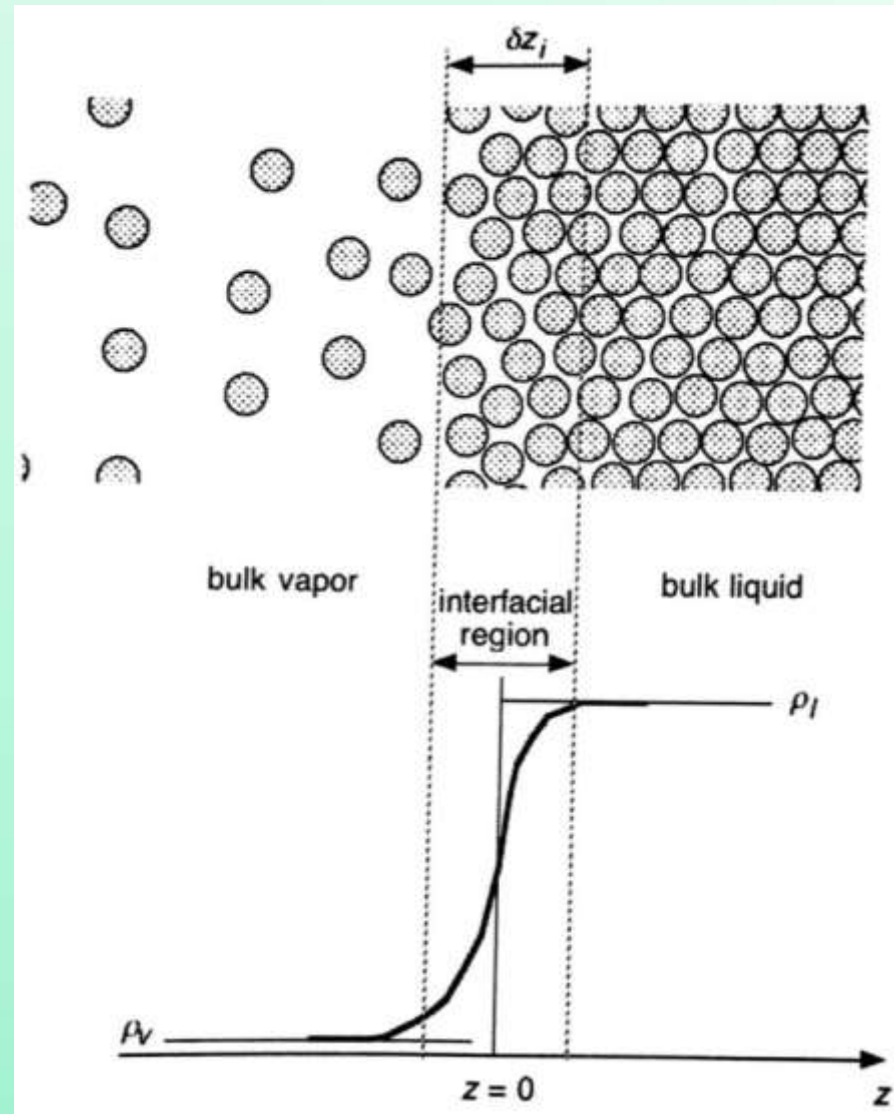
Postulate: The mean properties vary continuously across the transition region between the bulk phases:

Mean molar density,  $\hat{\rho} = \frac{\rho_n}{N_A}$

On the liquid side of the interfacial region,  $\rho$  is lower than that in the bulk liquid.

Energy per molecule in the interfacial region > bulk liquid.

The system has an additional free energy per unit area of interface due to the presence of the interface.



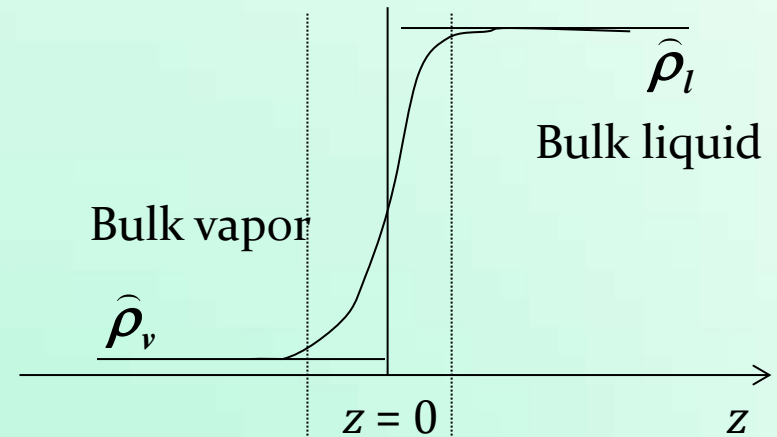
Variation of the molecular density

# Van der Waals (molecular) theory of capillarity

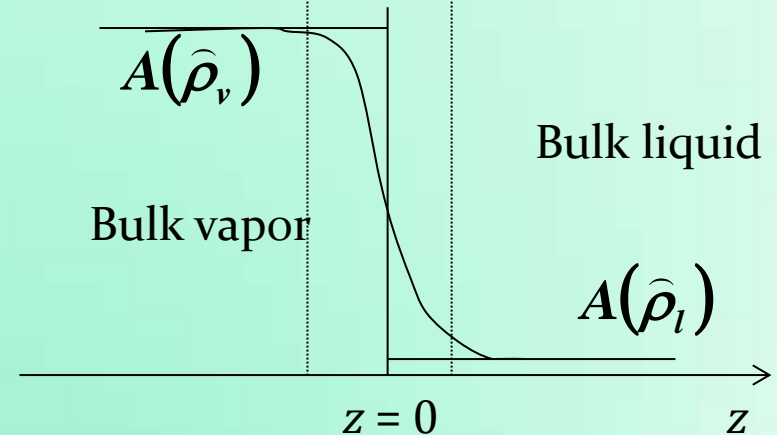
Interfacial tension is due to the excess interfacial free energy per unit area.

The interface surface is so chosen that the mass in the interfacial region with a distributed density profile is the same as would exist in the region with a discontinuous density step change at  $z = 0$ .

$$\int_{-\infty}^0 (\bar{\rho} - \hat{\rho}_v) dz + \int_0^{\infty} (\bar{\rho} - \hat{\rho}_l) dz = 0$$



Variation of mean molar density



Variation of the volumetric free energy

# Van der Waals (molecular) theory of capillarity

$$\sigma = \int_{-\infty}^0 [A - A(\hat{\rho}_v)] dz + \int_0^{\infty} [A - A(\hat{\rho}_l)] dz$$

$A$  is the Helmholtz free energy per unit volume

the maximum work a system can do @ constant  $V$  &  $T$

- Apply a system held at constant  $T$  with a  $V$  that encompasses the interfacial region over a unit area of the interface.
- Equilibrium corresponds to a minimum in volumetric free energy ( $A$ ).
- $\sigma$  is the property nothing but this equilibrium free energy.

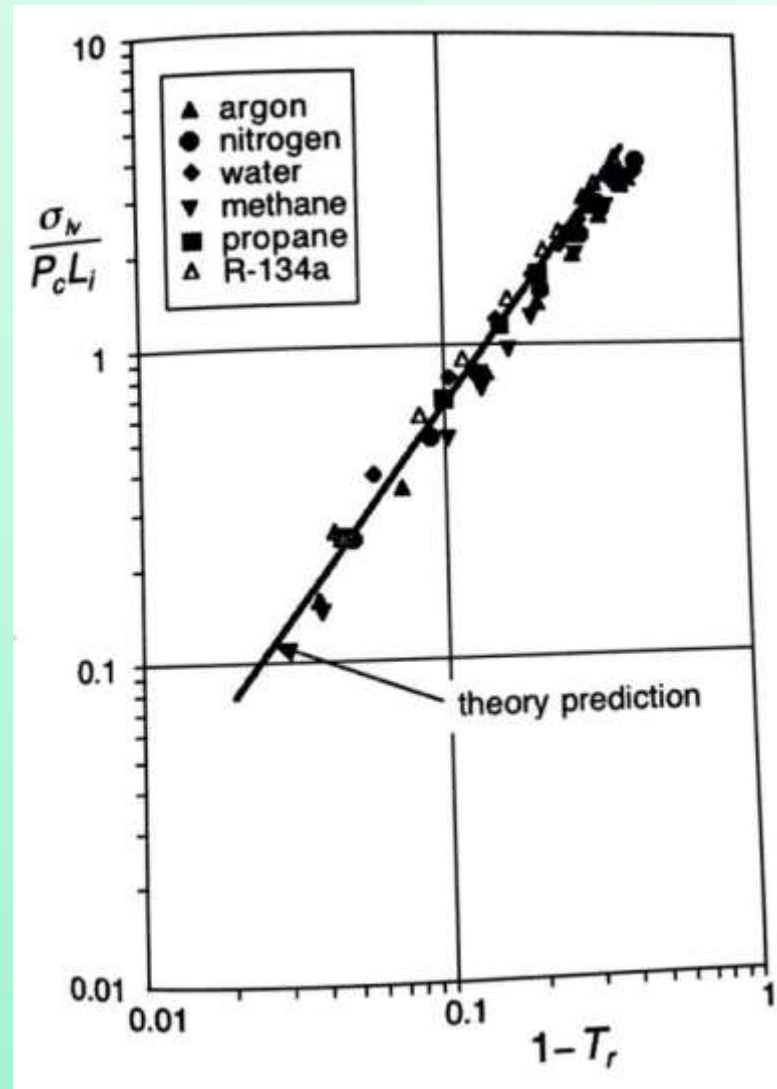
# Surface Tension – Molecular Theory of Capillarity

Modified Redlich-Kwong model for dimensionless  $\sigma$ :

$$\frac{\sigma_{lv}}{P_c L_i} = 14.65 \left( 1 - \frac{T}{T_c} \right)^{1.33}$$

$L_i$  is a characteristic length associated with the size of the interfacial region.

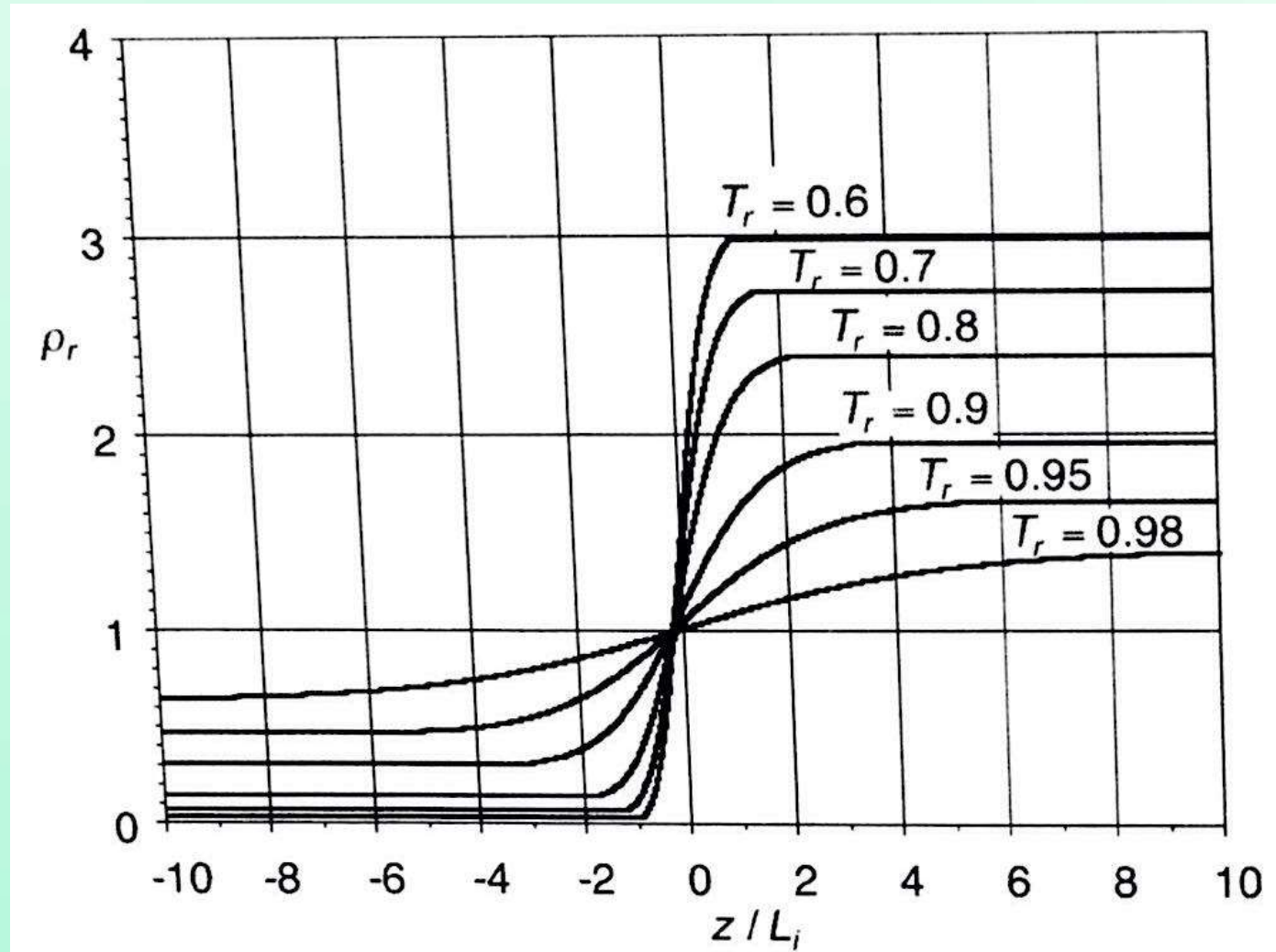
$$L_i = \left( \frac{k_B T_c}{P_c} \right)^{\frac{1}{3}}$$



# Interface properties

$$\rho_r = \frac{\rho}{\rho_c}$$

$$T_r = \frac{T}{T_c}$$



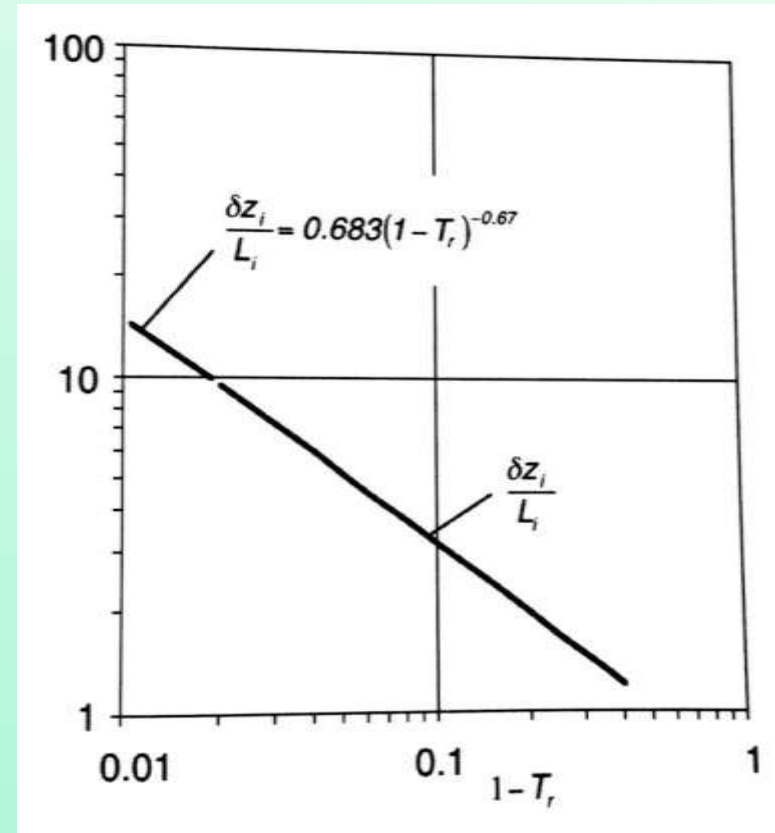
Reduced density profiles across the interfacial region predicted at various reduced temperatures

# Dimensionless Thickness of the Interfacial Region

Dimensionless interfacial thickness:

$$\frac{\delta z_i}{L_i} = 0.683 \left( 1 - \frac{T}{T_c} \right)^{-0.67}$$

	$T_c$ (K)	$P_c$ (MPa)	$L_i$ (nm)
N <sub>2</sub>	126.2	3.400	0.800
CH <sub>4</sub>	190.6	4.599	0.830
Ar	150.7	4.865	0.753
O <sub>2</sub>	154.5	5.043	0.751
H <sub>2</sub> O	647.3	22.129	0.739
NH <sub>3</sub>	405.6	11.290	0.793
C <sub>3</sub> H <sub>8</sub> (propane)	369.9	4.248	1.063
SF <sub>6</sub>	318.7	3.760	1.054
R-134a	374.3	4.059	1.084



$$L_i = \left( \frac{k_B T_c}{P_c} \right)^{\frac{1}{3}}$$

# Dimensionless Thickness of the Interfacial Region

For water at atmospheric pressure, estimate the surface tension and the interfacial region thickness.

$$T_c = 647.3 \text{ K}, P_c = 22.1 \text{ MPa and } L_i = 0.739 \text{ nm}$$

$$\frac{\sigma_{lv}}{P_c L_i} = 14.65 \left( 1 - \frac{T}{T_c} \right)^{1.33}$$

$$L_i = \left( \frac{k_B T_c}{P_c} \right)^{\frac{1}{3}} \quad k_B = 1.38 \times 10^{-23} \text{ m}^2 \text{kg/s}^2 \cdot \text{K}$$

$$\frac{\delta z_i}{L_i} = 0.683 \left( 1 - \frac{T}{T_c} \right)^{-0.67}$$

# Dimensionless Thickness of the Interfacial Region

For water at atmospheric pressure, estimate the surface tension and the interfacial region thickness.

$$T_c = 647.3 \text{ K}, P_c = 22.1 \text{ MPa and } L_i = 0.739 \text{ nm}$$

$$\sigma_{lv} = \mathbf{0.105 \text{ N/m}}$$

$$\delta z_i = \mathbf{0.766 \text{ nm}}$$

$$T = \mathbf{300 \text{ K}}$$

$$\sigma_{lv} = \mathbf{0.07635 \text{ N/m}}$$

$$\delta z_i = \mathbf{0.898 \text{ nm}}$$

$$T = \mathbf{373.14 \text{ K}}$$

Actual value,  $\sigma = 0.0712 \text{ N/m @ } T = 303.2 \text{ K}$

$$\sigma = 0.0589 \text{ N/m @ } T = 373.14 \text{ K}$$

Effective diameter of a water molecule = 0.28 nm

# **Multiphase Flow and Heat Transfer**

ME546

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# Vapor Pressure

At a given pressure, the temperature at which a pure substance changes phase is  $T_{sat}$ .

At a given temperature, the pressure at which a pure substance changes phase is  $P_{sat}$ .

**Vapor Pressure** ( $P_v$ ) of a pure substance is defined as the pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature.

For a pure substance,  $P_v$  is a Property and  $P_v = P_{sat}$ .

Temperature $T, ^\circ\text{C}$	Saturation Pressure $P_{sat}, \text{kPa}$
-10	0.260
-5	0.403
0	0.611
5	0.872
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.38
50	12.35
100	101.3 (1 atm)
150	475.8
200	1554
250	3973
300	8581

**Saturation/vapor pressure of water**

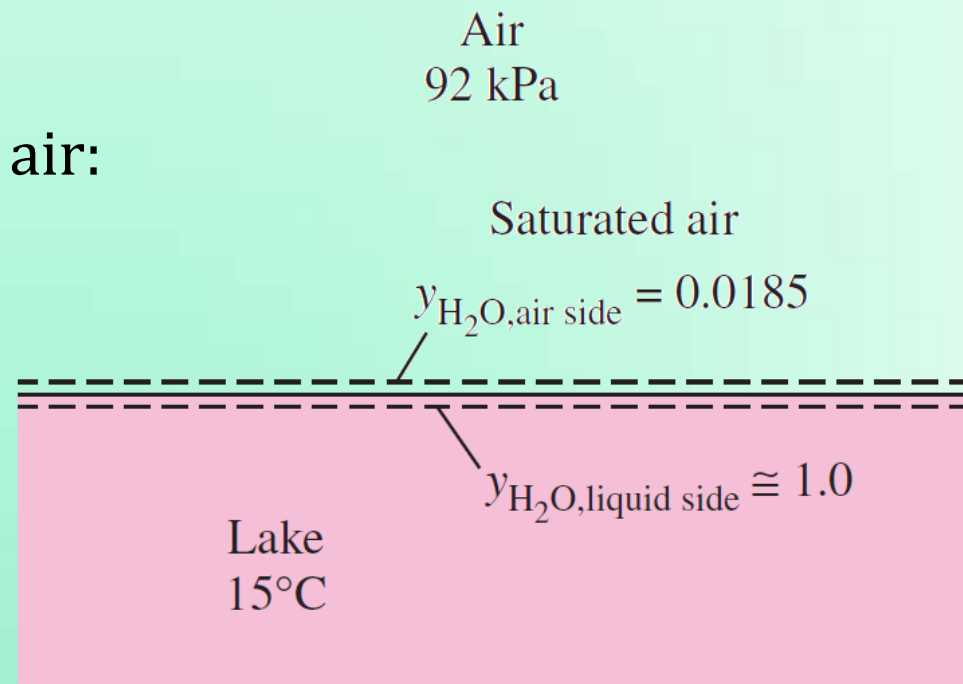
# Vapor Pressure

Determine the mole fraction of the water vapor at the surface of a lake whose temperature is 15°C. The atmospheric pressure at lake level is 92 kPa.

At phase equilibrium:  $P_v = P_{sat @ 15^\circ\text{C}} = 1.7057 \text{ kPa}$

Mole fraction of water vapor in air:

$$x_v = \frac{P_v}{P} = \frac{1.7057}{92} = 0.0185$$



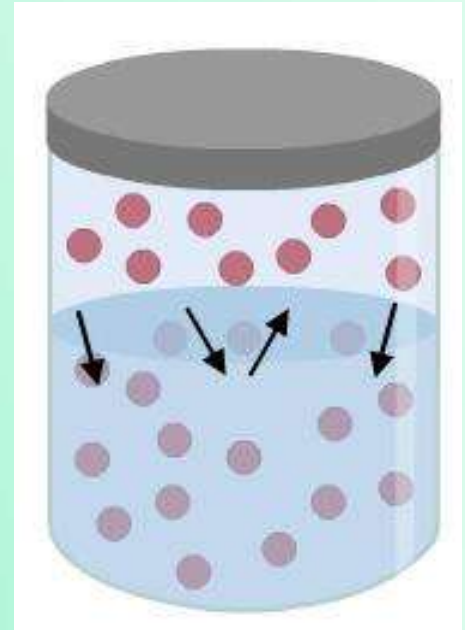
# Henry's Law

The amount of a gas that dissolves in a liquid is directly proportional to the partial pressure of that gas ( $P_{i,g}$ ) in equilibrium with that liquid.

Mole fraction of species  $i$  on liquid side,

$$x_{i,l} = \frac{P_{i,g}}{H}$$

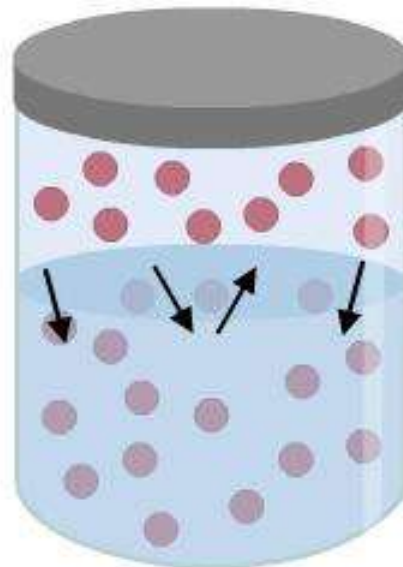
Henry's constant in pressure units (Pa)



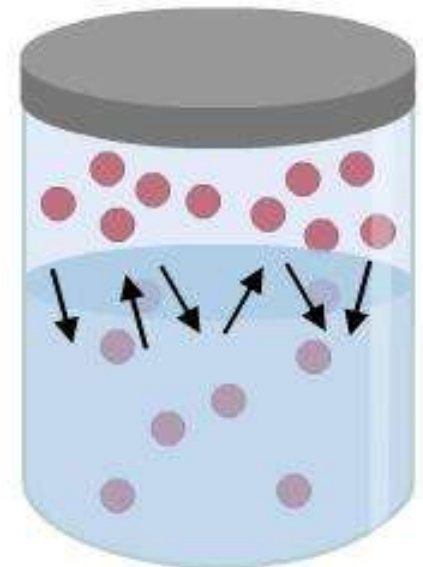
# Henry's Law

1. The concentration of a gas dissolved is  $\propto H^{-1}$
2.  $H \uparrow$  with  $T \uparrow$

$$x_{i,l} = \frac{P_{i,g}}{H}$$



Low temperature

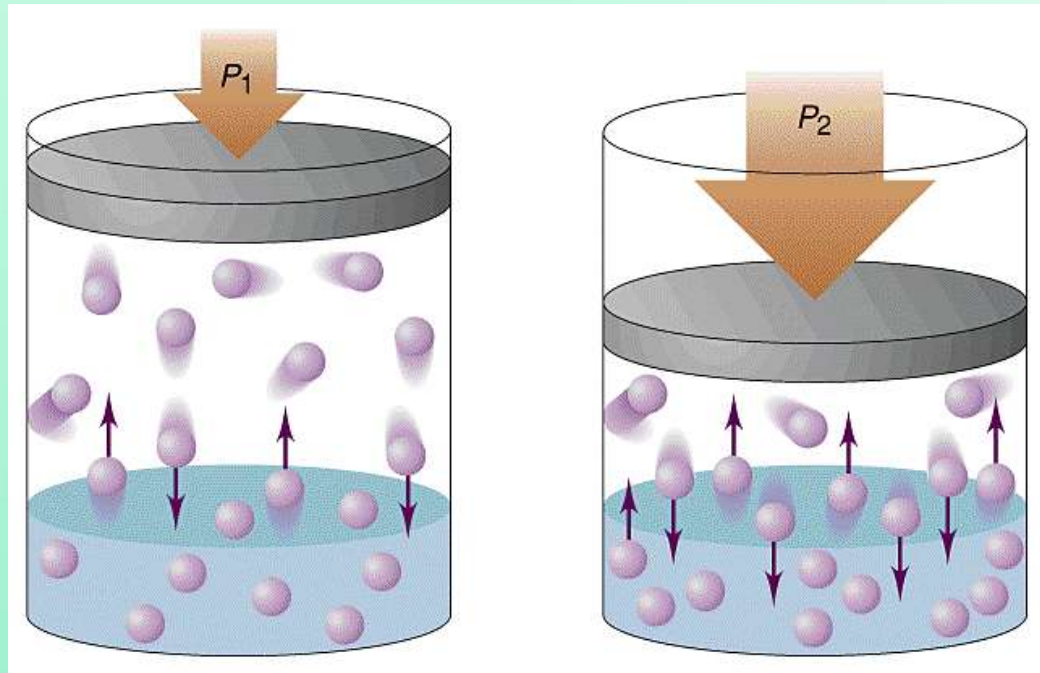


High temperature

# Henry's Law

1. The concentration of a gas dissolved is  $\propto H^{-1}$
2.  $H \uparrow$  with  $T \uparrow$
3.  $y_{i,l} \uparrow$  with  $P_i \uparrow$

$$x_{i,l} = \frac{P_{i,g}}{H}$$



# Henry's Law

$$x_{i,l} = \frac{P_{i,g}}{H}$$

Henry's constant  $H$  (in bars) for selected gases in water at low to moderate pressures (for gas  $i$ ,  $H = P_{i,\text{gas side}}/y_{i,\text{water side}}$ ) (from Mills, Table A.21, p. 874)

Solute	290 K	300 K	310 K	320 K	330 K	340 K
H <sub>2</sub> S	440	560	700	830	980	1140
CO <sub>2</sub>	1,280	1,710	2,170	2,720	3,220	—
O <sub>2</sub>	38,000	45,000	52,000	57,000	61,000	65,000
H <sub>2</sub>	67,000	72,000	75,000	76,000	77,000	76,000
CO	51,000	60,000	67,000	74,000	80,000	84,000
Air	62,000	74,000	84,000	92,000	99,000	104,000
N <sub>2</sub>	76,000	89,000	101,000	110,000	118,000	124,000

Determine the concentration of dissolved air in water at ambient conditions.  $H_{N_2} = 9.2 \times 10^9$  Pa,  $H_{O_2} = 4.3 \times 10^9$  Pa

At phase equilibrium:  $P_v = P_{sat @ 30^\circ\text{C}} = 4.25$  kPa

$$P_{total} = P_v + P_{dry-air}$$

$$P_{dry-air} = 101.42 - 4.25 = 97.17 \text{ kPa}$$

$$P_{dry-air} = P_{N_2} + P_{O_2}$$

$$P_{N_2} = 79\% P_{dry-air} = 76.7643 \text{ kPa}$$

$$\text{Mole fraction of } N_{2,l}, x_{N_{2,l}} = \frac{P_{N_2}}{H} = \frac{76.7643 \times 10^3}{9.2 \times 10^9} = 8.34 \times 10^{-6}$$

$$\text{Mass fraction of } N_{2,l}, y_{N_{2,l}} = x_{N_{2,l}} \times \frac{M_{N_2}}{M_{H_2O}} = 13 \times 10^{-6}$$

$$\text{Solubility of } N_2 \text{ in water, } S_{N_2} = 13 \times 10^{-3} \text{ g/kg}$$

$$P_{O_2} = 21\% P_{dry-air} = 20.4057 \text{ kPa}$$

$$\text{Mole fraction of } O_{2,l}, x_{O_{2,l}} = \frac{P_{O_2}}{H} = \frac{20.4057 \times 10^3}{4.3 \times 10^9} = 4.75 \times 10^{-6}$$

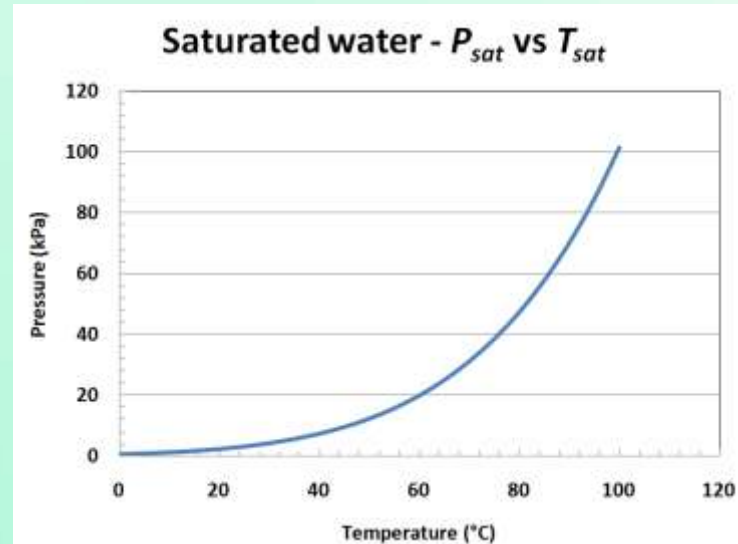
$$\text{Mass fraction of } O_{2,l}, y_{O_{2,l}} = x_{O_{2,l}} \times \frac{M_{O_2}}{M_{H_2O}} = 8.5 \times 10^{-6}$$

$$\text{Solubility of } O_2 \text{ in water, } S_{O_2} = 8.5 \times 10^{-3} \text{ g/kg}$$

*i.e.*, 8.4 ppm of dissolved  $O_2$  in water

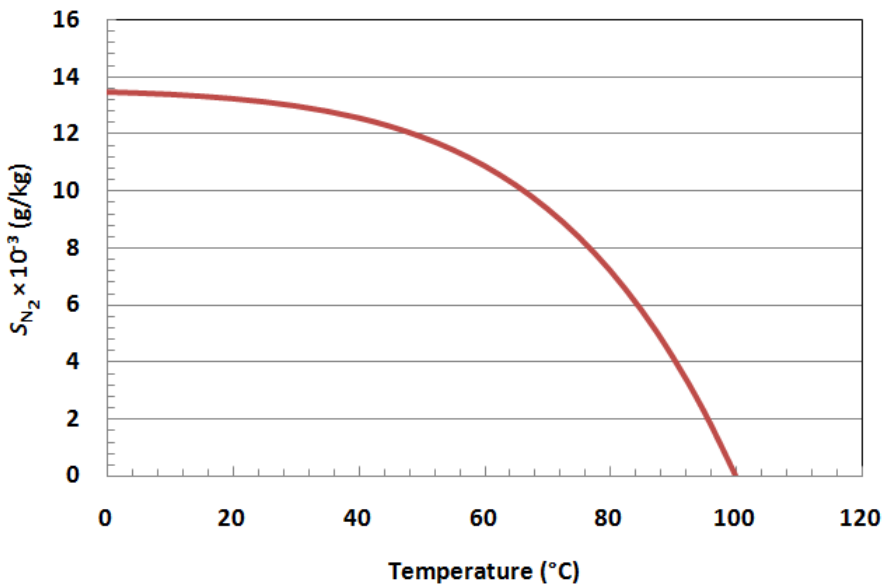
# Solubility with Temperature

$T$ (°C)	$P_{sat}$ (kPa)	$S_{N_2}$ (g/kg)	$S_{O_2}$ (g/kg)
30	4.247	$13 \times 10^{-3}$	$8.4 \times 10^{-3}$
50	12.325	$12 \times 10^{-3}$	$7.7 \times 10^{-3}$
70	31.202	$9.4 \times 10^{-3}$	$6.1 \times 10^{-3}$
80	47.416	$7.2 \times 10^{-3}$	$4.7 \times 10^{-3}$
90	70.183	$4.2 \times 10^{-3}$	$2.7 \times 10^{-3}$
100	101.42	0	0

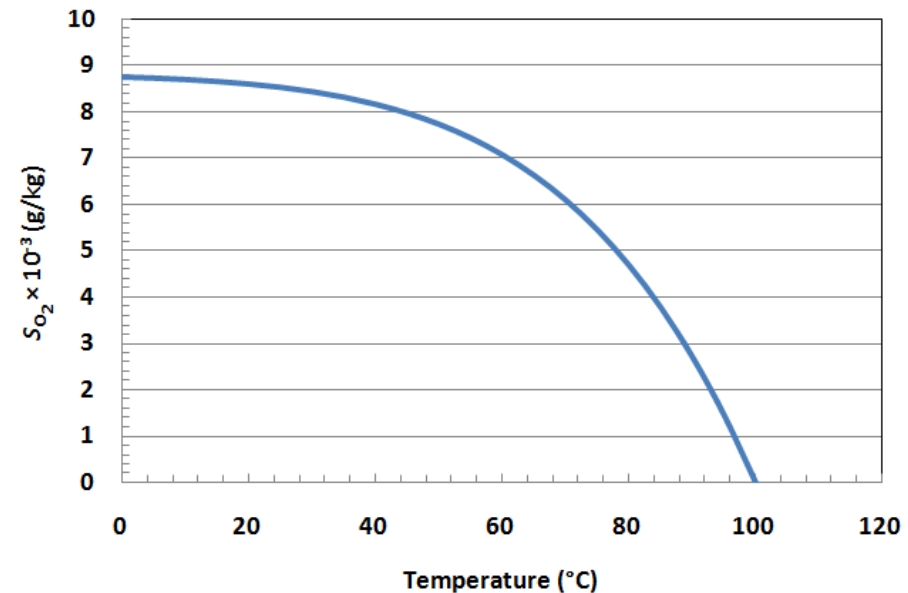


# Solubility of N<sub>2</sub> and O<sub>2</sub> at Water-Air Interface

Solubility of N<sub>2</sub> in water-air:  $T$  vs  $S_{N_2}$



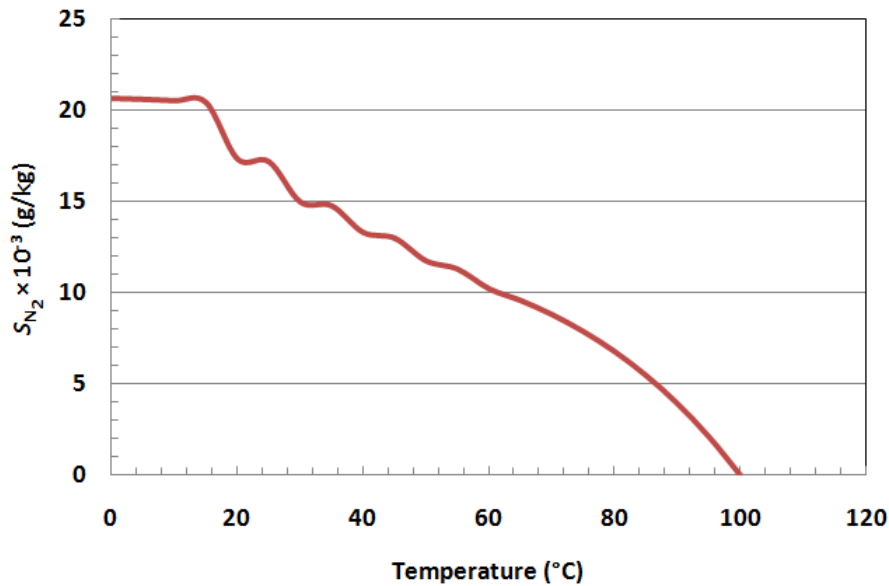
Solubility of O<sub>2</sub> in water-air:  $T$  vs  $S_{O_2}$



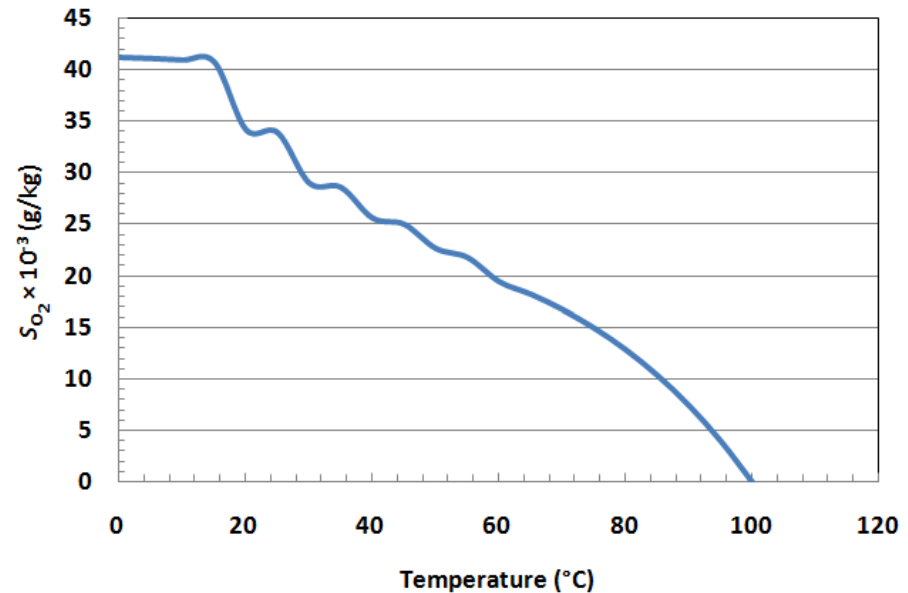
Henry's constant is considered not to vary with temperature

# Solubility of in Water

## Solubility of $N_2$ in water - $T$ vs $S_{N_2}$



## Solubility of $O_2$ in water - $T$ vs $S_{O_2}$



Henry's constant is considered to vary with temperature

# Degassing Techniques

1. By boiling or superheating
2. Vacuum degasification
3. Freeze-pump-thaw cycling
4. Membrane degasification

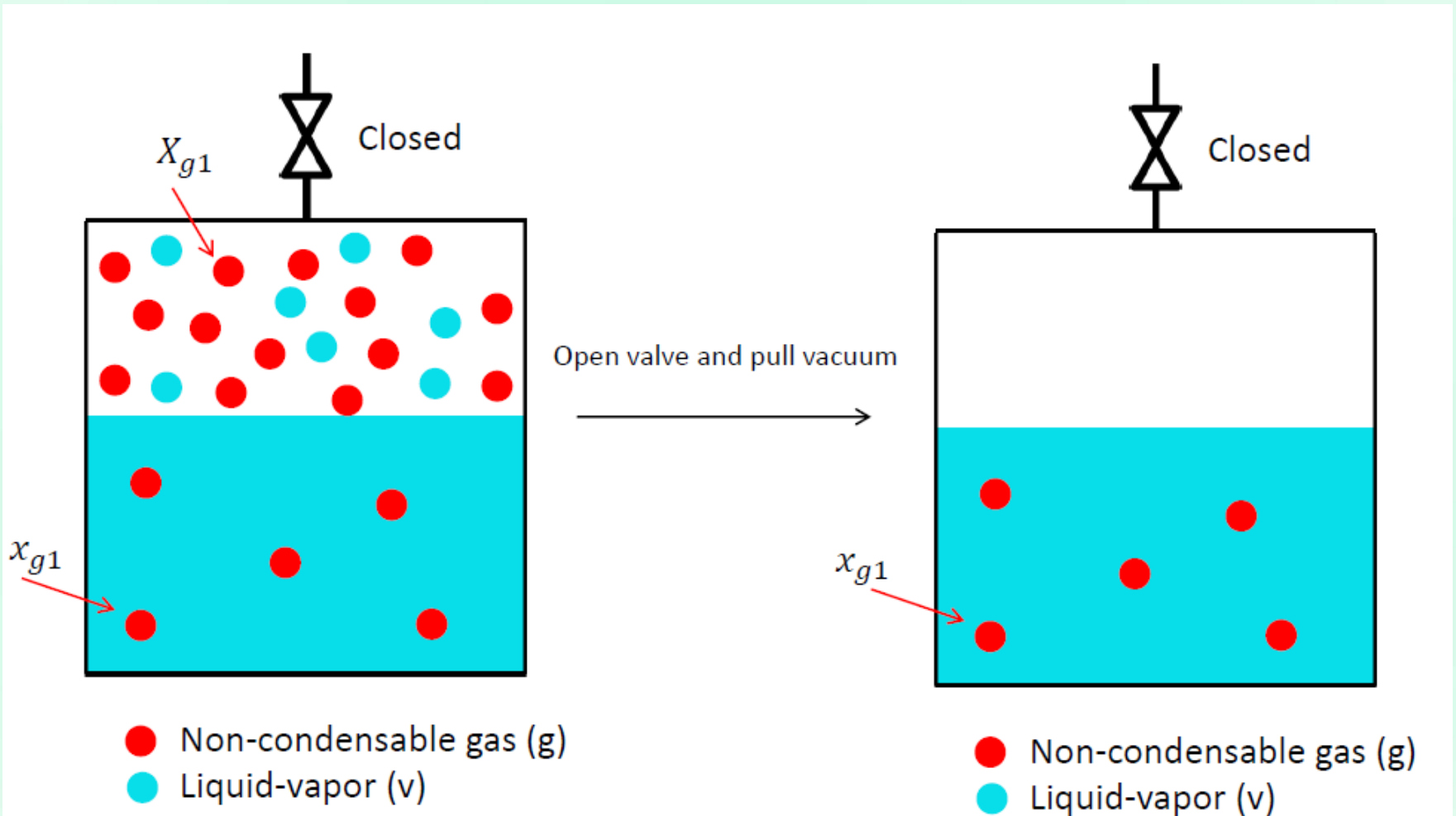
# Degassing Techniques

## **1. By boiling or superheating**

- Low purity limits
- Significant fluid is lost while pulling vacuum

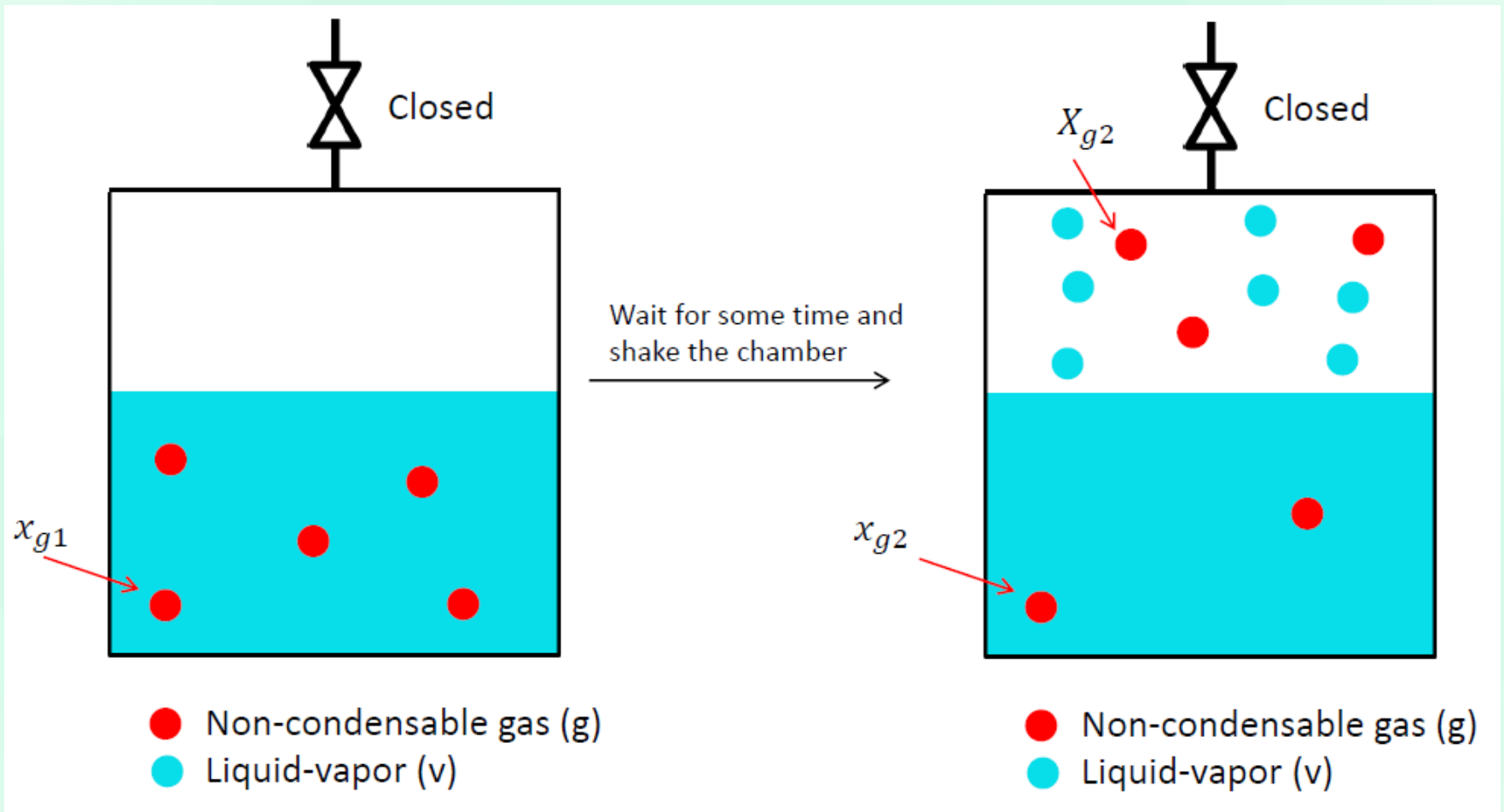
# Degassing Techniques

## 2. Vacuum degasification (Assuming only v & g are evacuated)



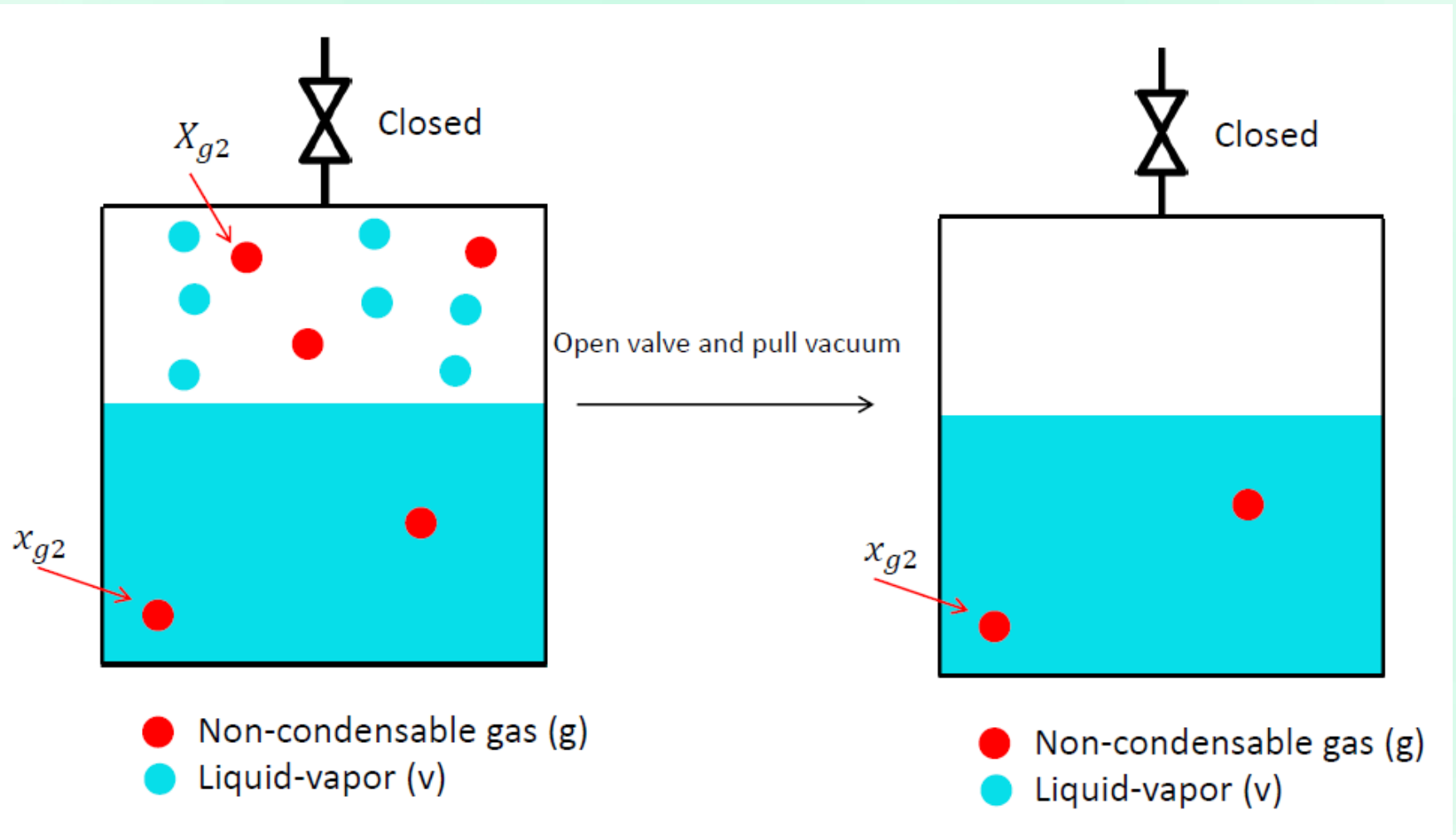
# Degassing Techniques

## 2. Vacuum degasification



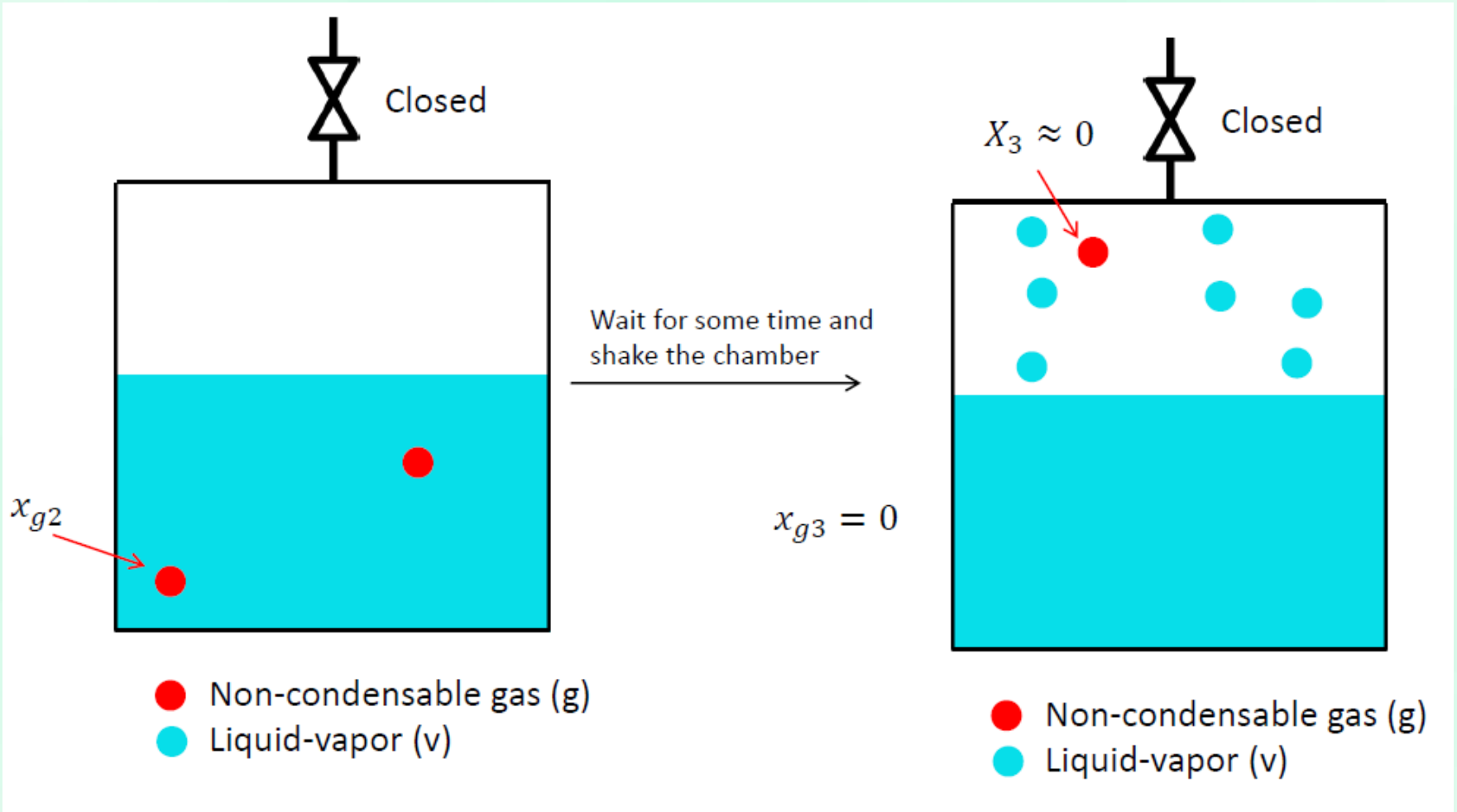
# Degassing Techniques

## 2. Vacuum degasification



# Degassing Techniques

## 2. Vacuum degasification

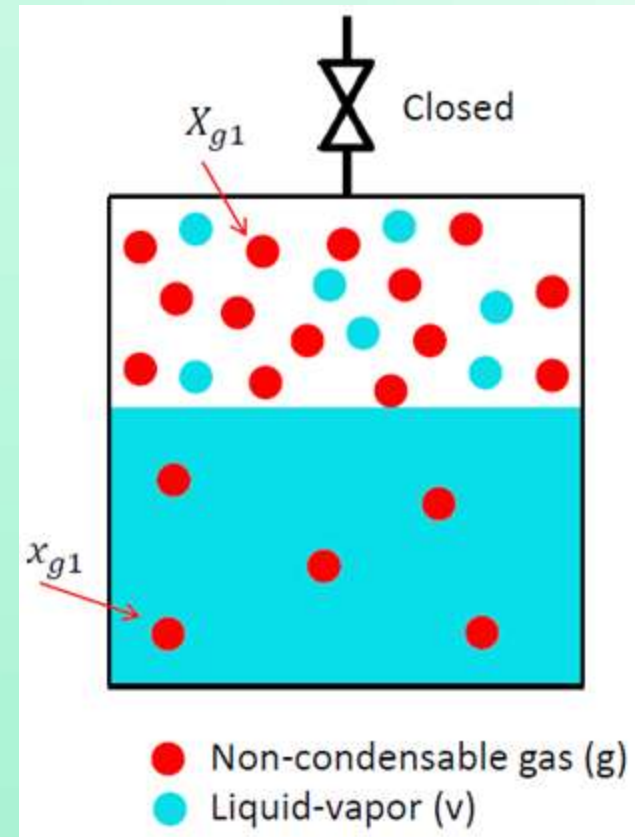


Imagine an insulated chamber with negligible thermal mass where the volume of the vapor and nitrogen space is equal to the volume of the liquid space (1 liter). Start with 70°C, and 101.325 kPa total pressure. Compute the number of moles of N<sub>2</sub> dissolved in the chamber after 3 cycles of vacuum degasification.  $H_{N_2} = 9.2 \times 10^9 \text{ Pa}$

$$P_{sat @ 70^\circ\text{C}} = 31.176 \text{ kPa}$$

**Hint:** Calculate the number of moles of N<sub>2</sub> and water on gas side and also on liquid side for each cycle.

Assume that no other gas exists other than N<sub>2</sub> and H<sub>2</sub>O.



## On gas/vapor side

At phase equilibrium:  $P_v = P_{sat @ 70^\circ\text{C}} = 31.176 \text{ kPa}$

$$P_{total} = P_v + P_{N_2}$$

$$P_{N_2} = 101.325 - 31.176 = 70.149 \text{ kPa}$$

By definition  $P_{N_2}$  is the pressure exerted if the whole volume is filled with  $N_2$ . Here the  $V$  is 1 liter =  $0.001 \text{ m}^3$ .

$$n_{N_2, v} = \frac{P_{N_2} V}{RT} = \frac{70149 \times 0.001}{8.314 \times 343.14} = 0.0246 \text{ moles}$$

Similarly for water in vapor state:

$$n_{H_2O, v} = \frac{P_{H_2O} V}{RT} = \frac{31176 \times 0.001}{8.314 \times 343.14} = 0.0109 \text{ moles}$$

## On liquid side

Number of moles of H<sub>2</sub>O in 1 liter of volume:

$$n_{\text{H}_2\text{O},l} = \frac{V_{\text{H}_2\text{O},l}}{M_{\text{H}_2\text{O}}} = \frac{1 \text{ kg}}{18 \frac{\text{g}}{\text{mol}}} = 55.55 \text{ moles}$$

From Henry's law, mole fraction of N<sub>2</sub> in liquid water:

$$x_{\text{N}_2,l} = \frac{P_{\text{N}_2}}{H_{\text{N}_2}} = \frac{70149}{9.2 \times 10^9} = 7.62 \times 10^{-6}$$

By the definition of mole fraction:

$$x_{\text{N}_2,l} = \frac{n_{\text{N}_2,l}}{n_{\text{H}_2\text{O},l} + n_{\text{N}_2,l}} \Rightarrow n_{\text{N}_2,l} = n_{\text{H}_2\text{O},l} x_{\text{N}_2,l} = 0.424 \times 10^{-3} \text{ moles}$$

**We started with a chamber  
containing:**

$$n_{\text{H}_2\text{O},l} = 55.55 \text{ moles}$$

$$n_{\text{N}_2,l} = 0.424 \times 10^{-3} \text{ moles}$$

$$n_{\text{H}_2\text{O},v} = 0.0109 \text{ moles}$$

$$n_{\text{N}_2,v} = 0.0246 \text{ moles}$$

During the first cycle of vacuum degasification, we removed all the vapor, *i.e.*, 0.0109 moles of H<sub>2</sub>O and 0.0246 moles of N<sub>2</sub> from the chamber.

Number of molecules left in the chamber are:

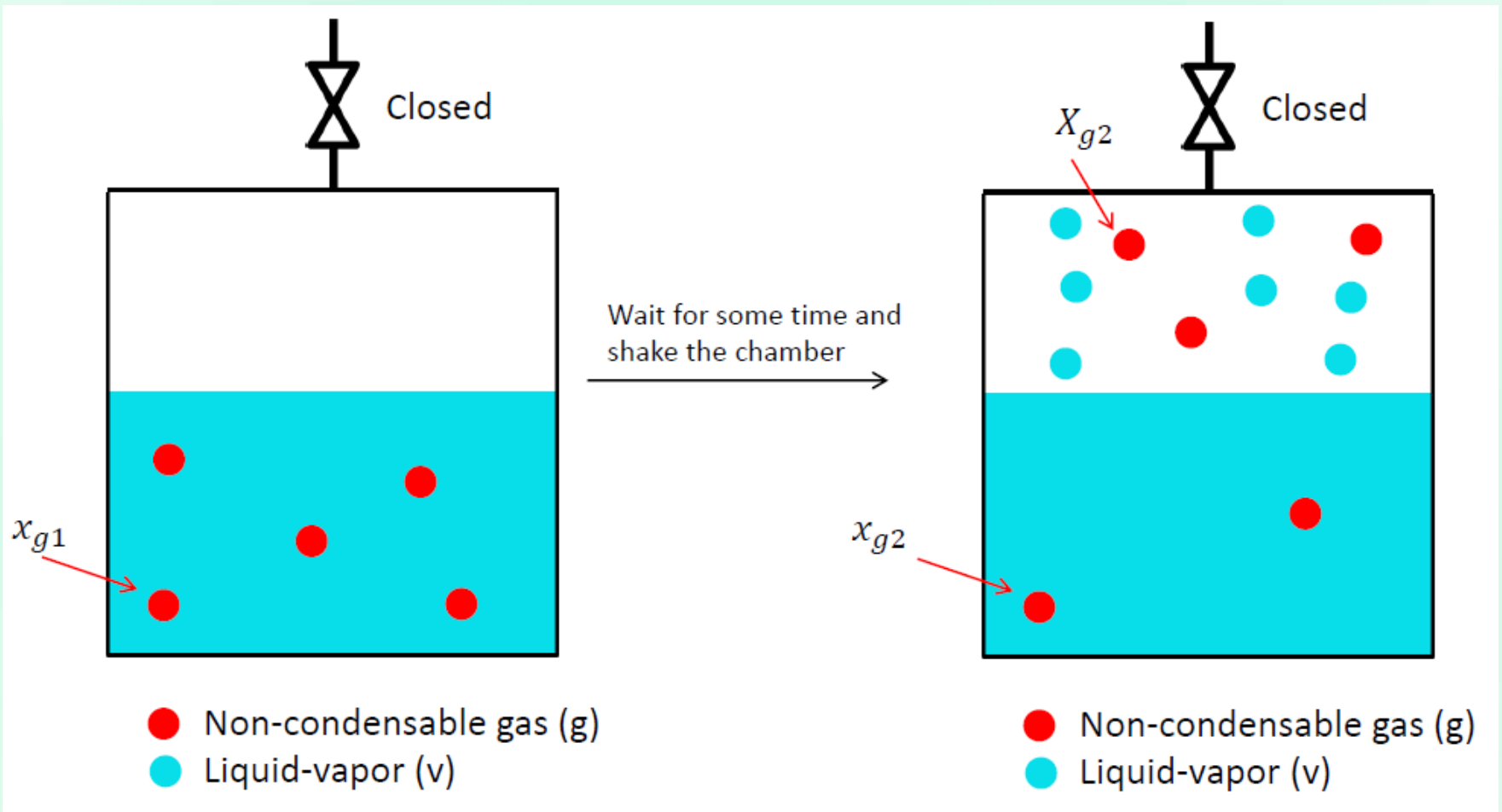
55.55 moles of H<sub>2</sub>O and 0.424 10<sup>-3</sup> moles of N<sub>2</sub>.

The chamber is now allowed to settle:

Phase equilibrium: Part of H<sub>2</sub>O (*l*) evaporates

Dissolved N<sub>2</sub> in H<sub>2</sub>O (*l*) comes-out following Henry's law.

# The molecules in $l$ have to be distributed in $l$ and $g$



## After First Cycle

At phase equilibrium:  $P_v = P_{sat @ 70^\circ\text{C}} = 31.176 \text{ kPa}$

55.55 moles of  $\text{H}_2\text{O}$  has to be distributed as  $n_{\text{H}_2\text{O},l}$  and  $n_{\text{H}_2\text{O},v}$

$\because n_{\text{H}_2\text{O},l} \ll n_{\text{H}_2\text{O},v}$ , Change in volume of  $\text{H}_2\text{O}$  ( $l$ ) is negligible,

*i.e.*,  $V_{\text{H}_2\text{O},v} = 1 \text{ liter} = 1 \text{ kg} = 0.001 \text{ m}^3$ .

$$\begin{array}{l} \Rightarrow \\ \text{Similar to previous calculation} \end{array} \left| \begin{array}{l} n_{\text{H}_2\text{O},l} = 55.55 \text{ moles} \\ n_{\text{H}_2\text{O},v} = 0.0109 \text{ moles} \end{array} \right.$$

*In a strict sense, they should be computed iteratively*

$P_{\text{N}_2}$  and  $P_{total}$  are not known at this stage.

# First Cycle

$$x_{\text{N}_2,l} = \frac{P_{\text{N}_2}}{H_{\text{N}_2}} \quad P_{\text{N}_2} = \frac{n_{\text{N}_2,v} RT}{V}$$

$$x_{\text{N}_2,l} = \frac{n_{\text{N}_2,l}}{n_{\text{H}_2\text{O},l} + n_{\text{N}_2,l}} \Rightarrow n_{\text{N}_2,l} = n_{\text{H}_2\text{O},l} x_{\text{N}_2,l}$$

$$n_{\text{N}_2,l} = \frac{n_{\text{H}_2\text{O},l} RT}{H_{\text{N}_2} V} n_{\text{N}_2,v}$$

$0.424 \times 10^{-3}$  moles of  $\text{N}_2$  is distributed as  $n_{\text{N}_2,l}$  and  $n_{\text{N}_2,v}$

$$n_{\text{N}_2,l} + n_{\text{N}_2,v} = 0.424 \times 10^{-3} \text{ moles}$$

## End of First Cycle

$$n_{\text{H}_2\text{O},l} = 55.55 \text{ moles}$$

$$n_{\text{N}_2,l} = 7.18 \times 10^{-6} \text{ moles}$$

$$n_{\text{H}_2\text{O},v} = 0.0109 \text{ moles}$$

$$n_{\text{N}_2,v} = 0.0004168 \text{ moles}$$

We started with

$$n_{\text{N}_2,l} = 0.424 \times 10^{-3} \text{ moles}$$

$$n_{\text{N}_2,v} = 0.0246 \text{ moles}$$

In one cycle of vacuum degasification, we observe reduction by 59 times of dissolved  $\text{N}_2$  in liquid  $\text{H}_2\text{O}$ .

## End of Second Cycle

$$n_{\text{H}_2\text{O},l} = 55.55 \text{ moles}$$

$$n_{\text{H}_2\text{O},v} = 0.0109 \text{ moles}$$

$$n_{\text{N}_2,l} = ??? \text{ moles}$$

$$n_{\text{N}_2,v} = ??? \text{ moles}$$

We started with

$$n_{\text{N}_2,l} = 0.424 \times 10^{-3} \text{ moles}$$

$$n_{\text{N}_2,v} = 0.0246 \text{ moles}$$

In two cycles of vacuum degasification, we observe reduction by ??? times of dissolved  $\text{N}_2$  in liquid  $\text{H}_2\text{O}$ .

## End of Third Cycle

$$n_{\text{H}_2\text{O},l} = 55.55 \text{ moles}$$

$$n_{\text{H}_2\text{O},v} = 0.0109 \text{ moles}$$

$$n_{\text{N}_2,l} = ??? \text{ moles}$$

$$n_{\text{N}_2,v} = ??? \text{ moles}$$

We started with

$$n_{\text{N}_2,l} = 0.424 \times 10^{-3} \text{ moles}$$

$$n_{\text{N}_2,v} = 0.0246 \text{ moles}$$

In three cycles of vacuum degasification, we observe reduction by ??? times of dissolved  $\text{N}_2$  in liquid  $\text{H}_2\text{O}$ .

# Degassing Techniques

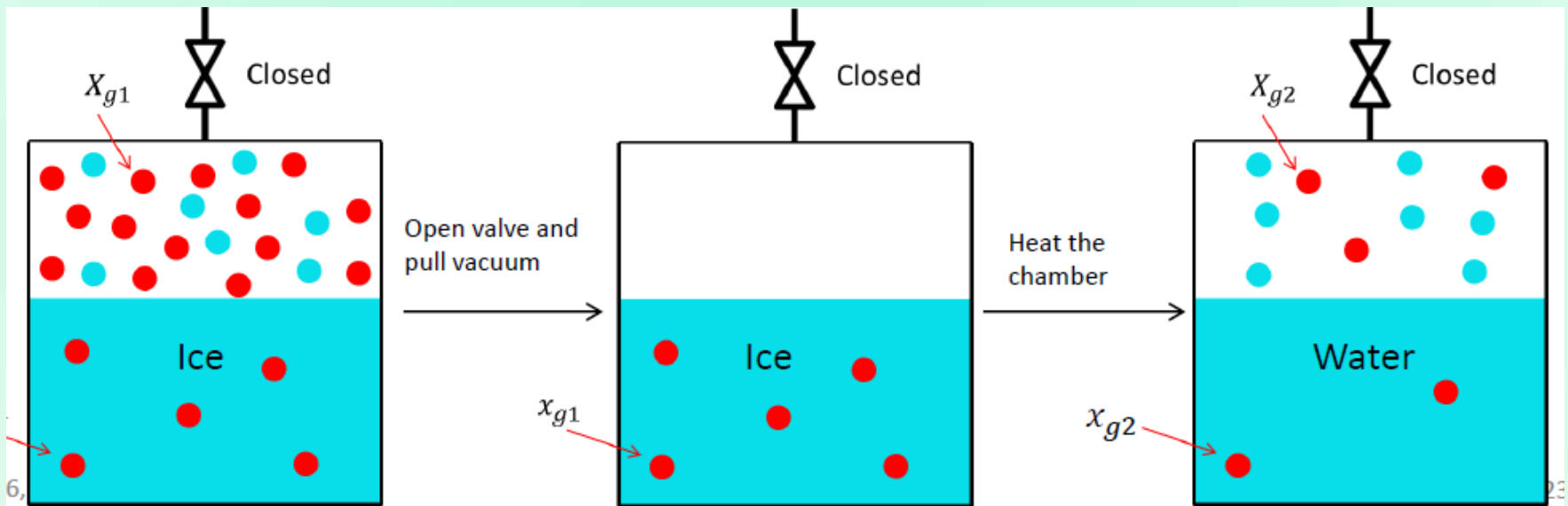
## 2. Vacuum degasification

- Some liquid is lost while pulling vacuum
- The chamber temperature needs to be controlled
- Needs 3 cycles to get high purity limits

# Degassing Techniques

## 3. Freeze-pump-thaw cycling

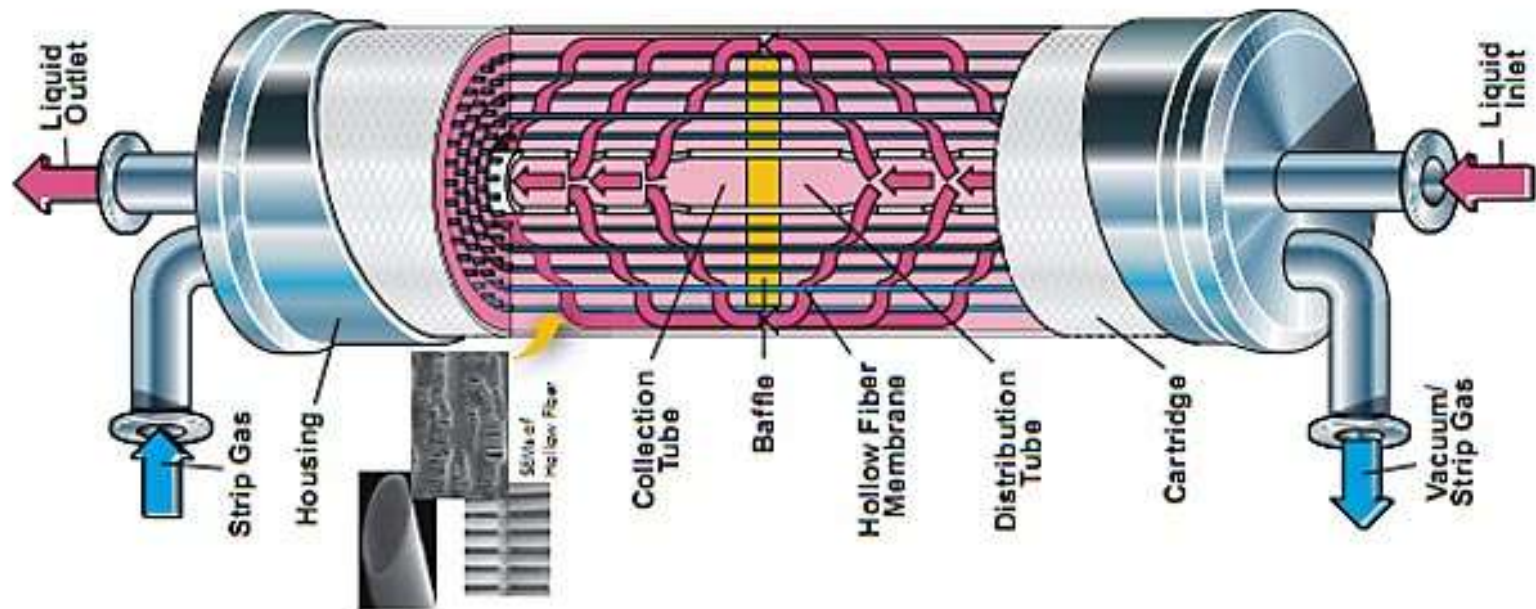
- Very small amount of fluid is lost
- Needs 3 cycles to get high purity limits
- Less hazardous



# Degassing Techniques

## 4. Membrane degasification

Pull vacuum through  
a membrane such as Gore-Tex



# Evaporation

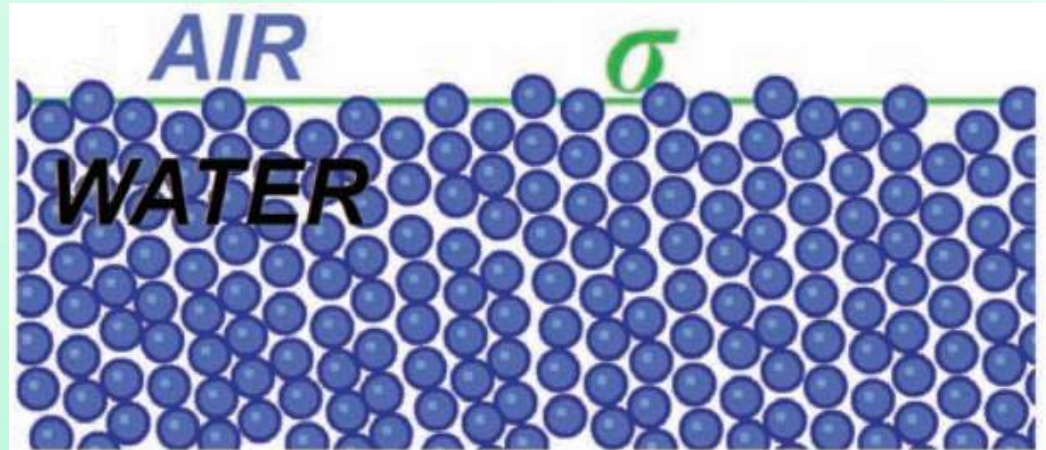
# **Multiphase Flow and Heat Transfer**

ME546

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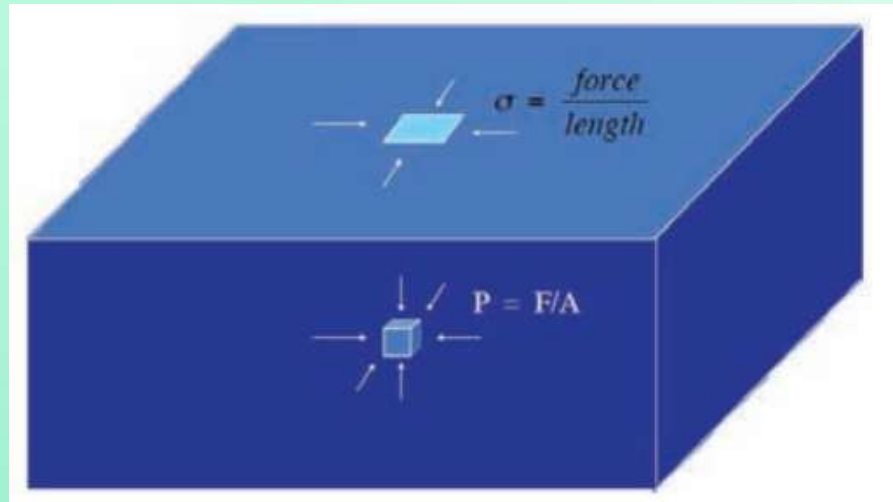
# Surface Tension

The free surface between air and water at a molecular scale

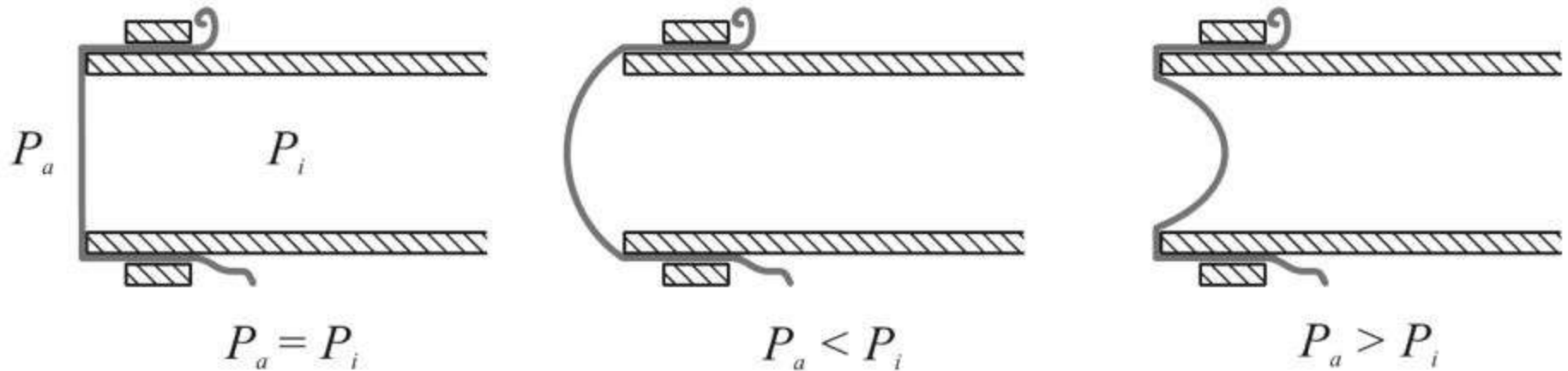


Molecules sitting at a free liquid surface against vacuum or gas have weaker binding than molecules in the bulk.

Surface tension is analogous to a negative surface pressure



# Surface Tension

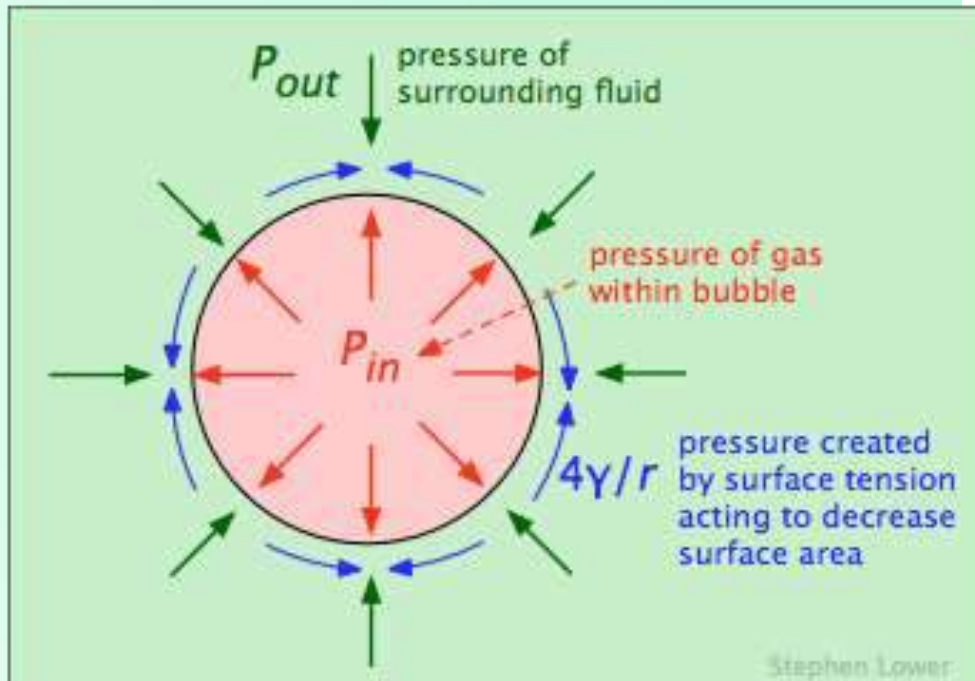
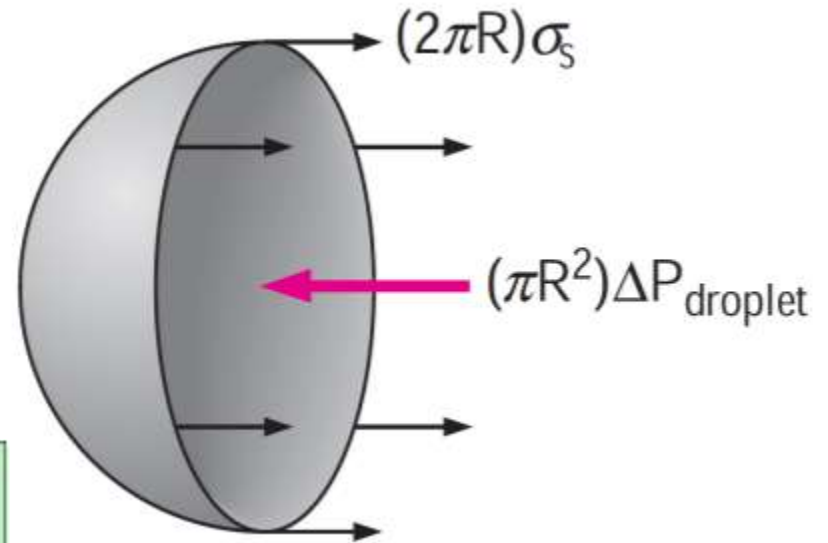


## Rubber membrane at the end of a cylindrical tube

An inner pressure  $P_i$  can be applied, which is different than the outside pressure  $P_a$ .

# Young-Laplace Equation

Surface tension acts along the circumference and the pressure acts on the area, horizontal force balances for the droplet.



# Young-Laplace Equation

Consider a differential increase in the radius of the droplet due to the addition of a differential amount of mass.

Surface tension is the increase in the surface energy per unit area.

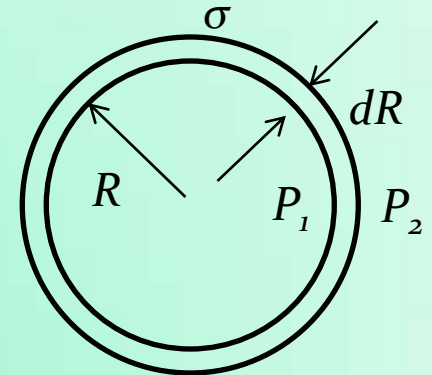
The increase in the surface energy of the droplet during the differential expansion process:

$$dW_{\text{surface}} = \sigma dA$$

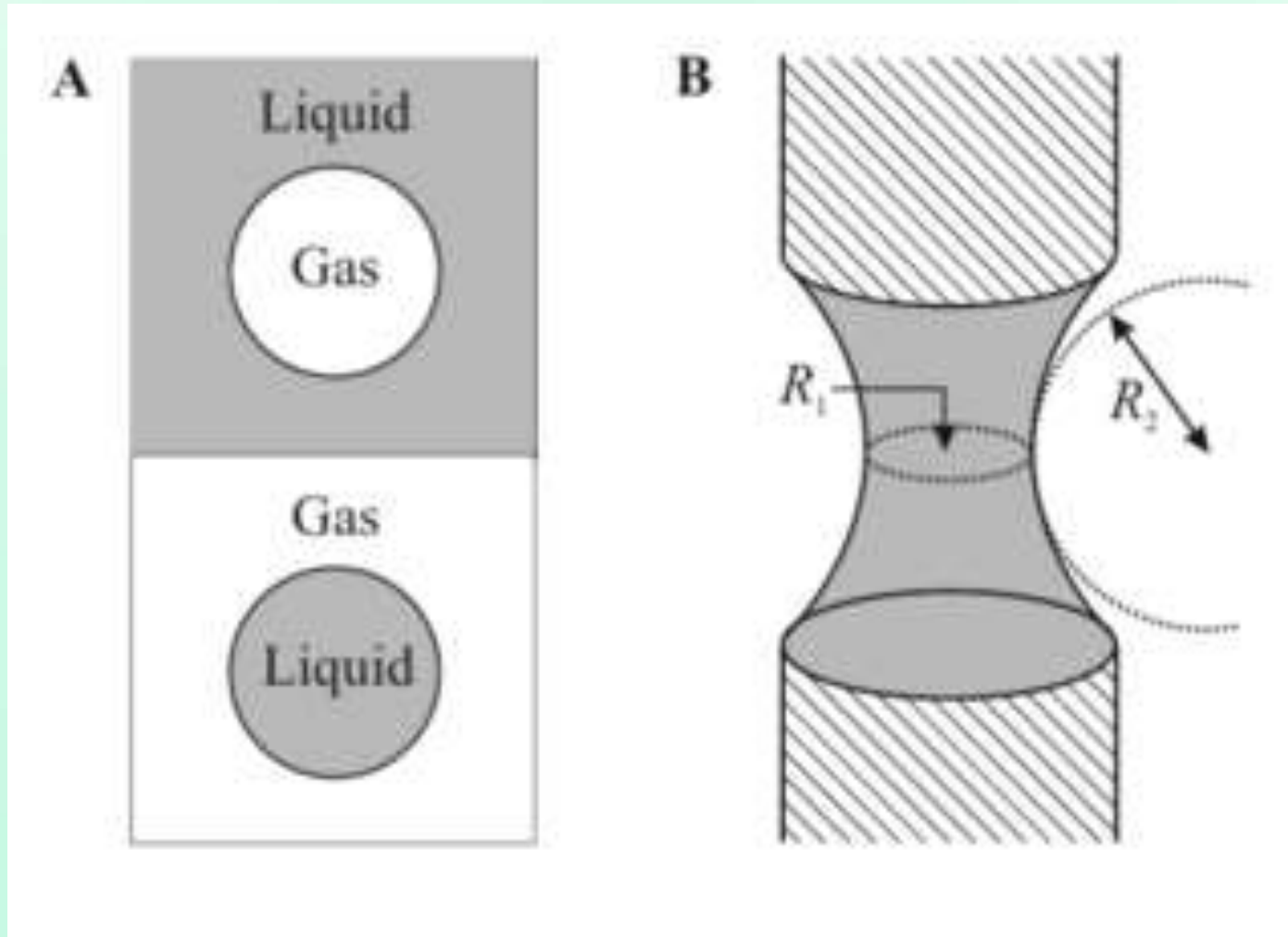
$$\delta W_{\text{expansion}} = FdR = \Delta P AdR = \Delta P dV$$

$$\Delta P = P_1 - P_2 = \frac{2\sigma}{R}$$

$P_1 - P_2$  is positive and so  $P_1 > P_2$



# Young-Laplace Equation



A liquid meniscus with radii of curvature of opposite sign between two solid cylinders

# Young-Laplace Equation

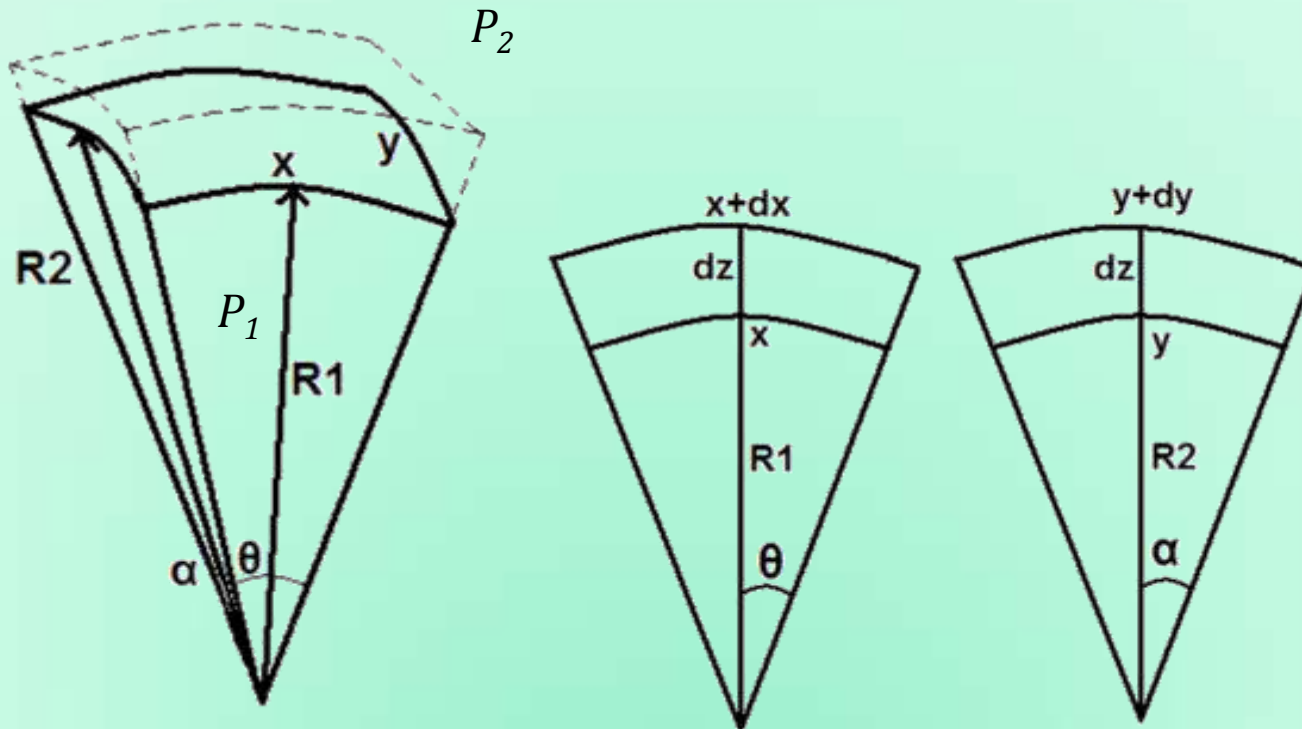
$$dA = (x + dx)(y + dy) - xy = xdy + ydx$$

$$dW_{\text{surface}} = \sigma dA = \sigma(xdy + ydx)$$

$$dW_{\text{expansion}} = \Delta P Adz = \Delta Pxydz$$

$$\Delta P = P_1 - P_2$$

$P_1$  is on concave side



# Young-Laplace Equation

$$\Delta P \, xy \, dz = \sigma (x \, dy + y \, dx)$$

$$\frac{x + dx}{x} = \frac{R_1 + dz}{R_1}$$

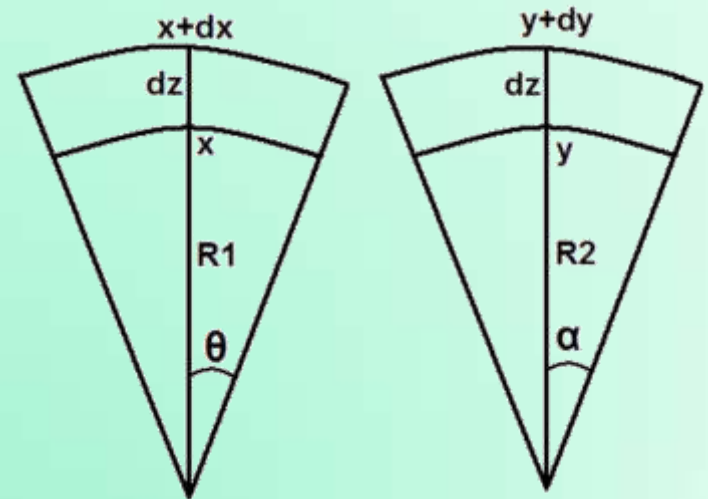
$$\frac{y + dy}{y} = \frac{R_2 + dz}{R_2}$$

$$\Delta P = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

Capillary pressure difference,

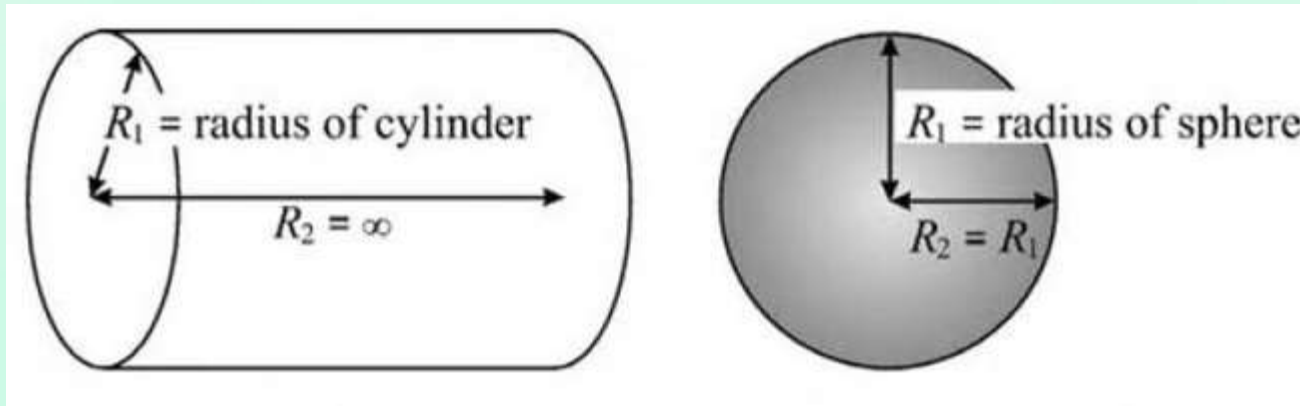
$$\Delta P = P_1 - P_2$$

Pressure on the concave side is higher



# Young-Laplace Equation

The planes defining the radii of curvature must be perpendicular to each other and contain the surface normal.



For a cylinder of radius  $R$  a convenient choice is

$$R_1 = R \text{ and } R_2 = \infty \text{ so that the curvature is } 1/R.$$

For a sphere with radius  $R$  we have

$$R_1 = R_2 \text{ and the curvature is } 2/R.$$

# Young-Laplace Equation

Compare a spherical bubble with a diameter of 1 mm and a bubble of 10 nm diameter in pure water.  $\sigma = 0.072 \text{ N/m}$

$$\Delta P_{1\text{mm}} = 288 \text{ Pa}$$

$$\Delta P_{1\text{nm}} = 288 \text{ MPa}$$

$$P_{\text{inside, 1nm}} = 288 \text{ MPa} + 0.1 \text{ MPa} = 288.1 \text{ MPa}$$

# Young-Laplace Equation

If we know the shape of a liquid surface we know its curvature and we can calculate the pressure difference.

In the absence of external fields (e.g., gravity), the pressure is same everywhere in the liquid; otherwise there would be a flow of liquid of regions of low pressure.

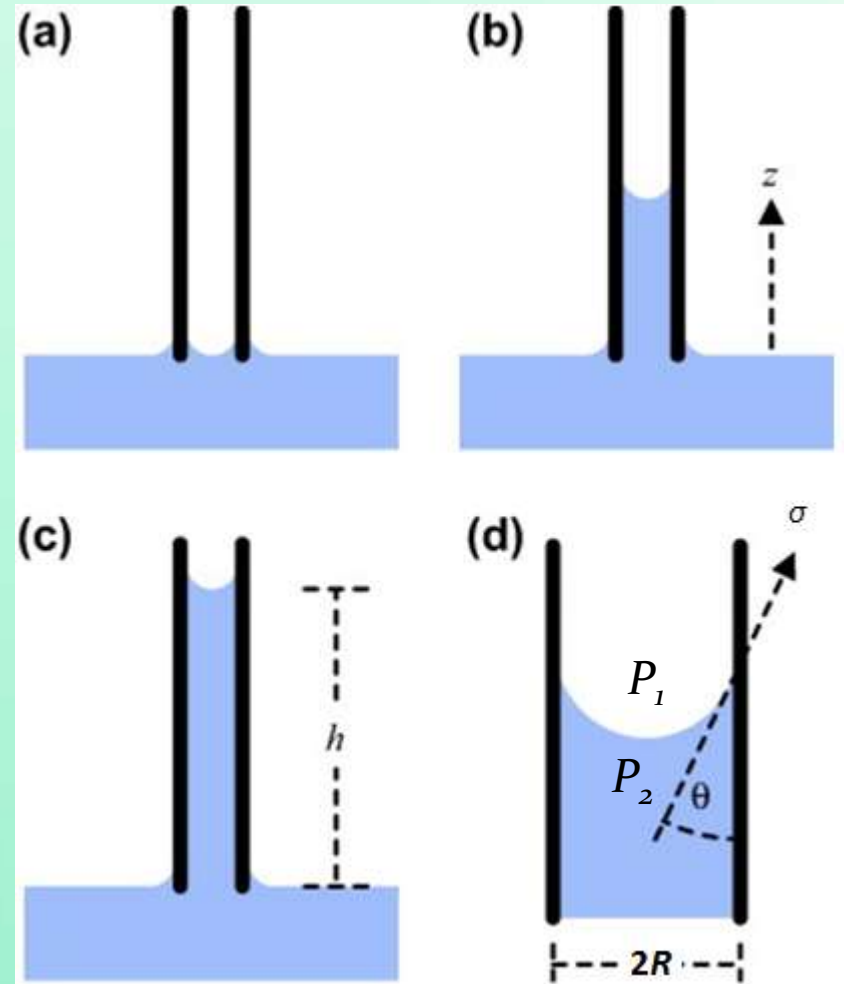
Thus,  $\Delta P$  is constant and Young-Laplace equation tells us in this case the surface of the liquid has the same curvature everywhere.

It is possible to calculate the equilibrium shape of a liquid surface.

If we know the pressure difference and some boundary conditions (such as volume of the liquid and its contact line) we can calculate the geometry of the liquid surface.

# Capillary Rise or Depression

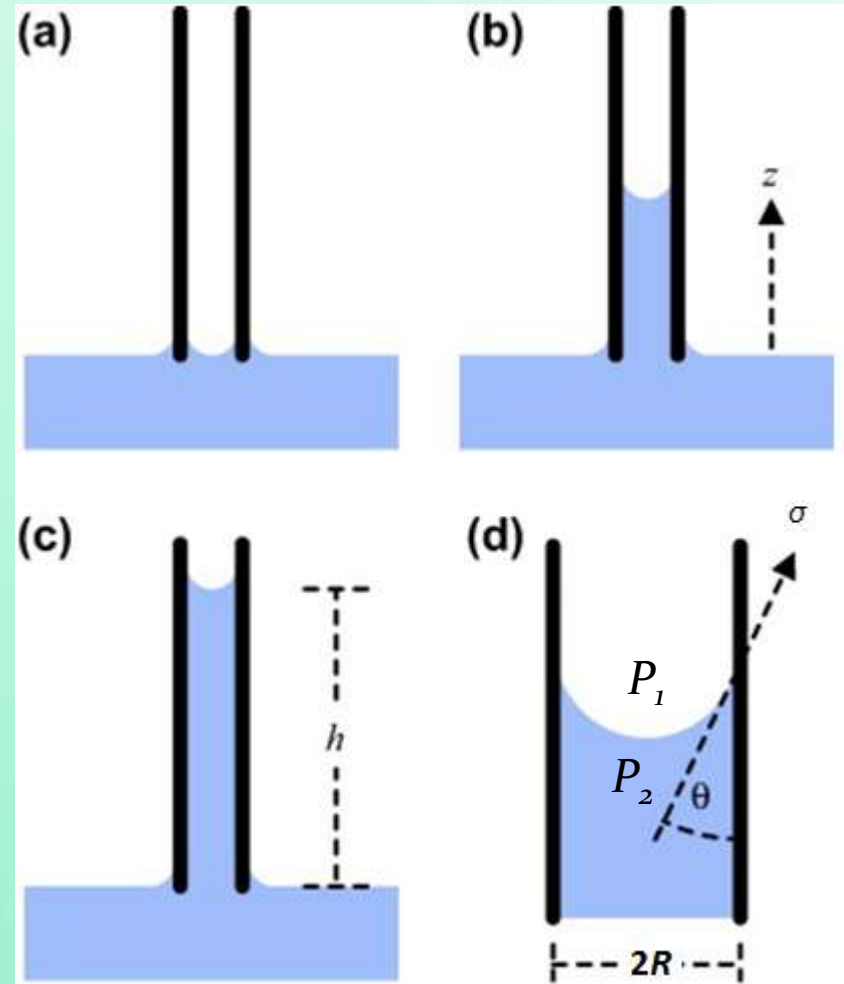
Why makes the liquid column to rise/fall?



# Capillary Rise or Depression

Why makes the liquid column to rise/fall?

To satisfy Young-Laplace equation.



# Capillary Rise or Depression

Why makes the liquid column to rise/fall?

To satisfy Young-Laplace equation.

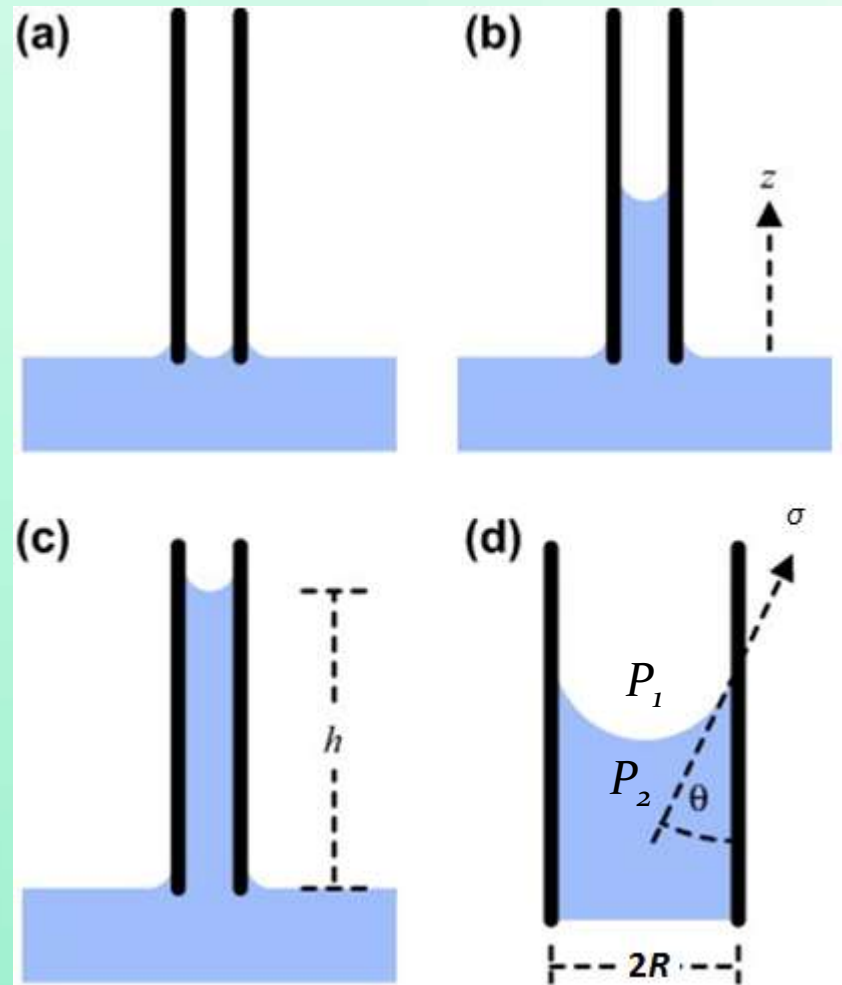
Let us assume that the meniscus is of spherical of radius,  $a$ .

$$P_1 - P_2 = \frac{2\sigma}{a}$$

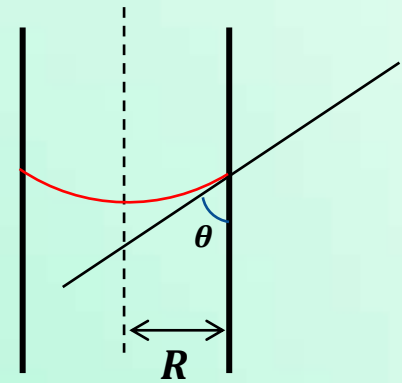
From hydrostatics:

$$P_2 = P_1 + \rho_v gh - \rho_l gh$$

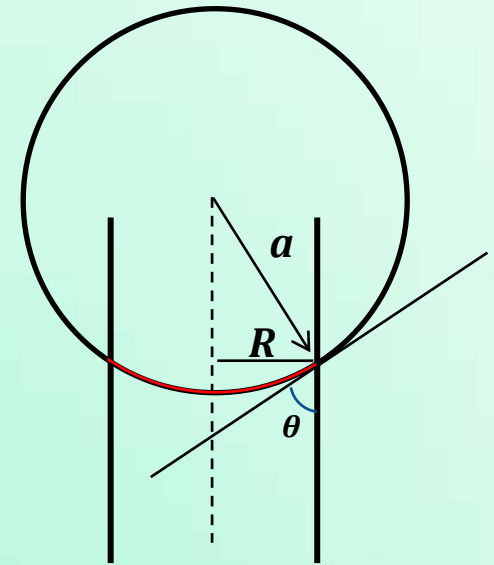
$$P_1 - P_2 = (\rho_l - \rho_v)gh$$



# Capillary Rise or Depression



# Capillary Rise or Depression

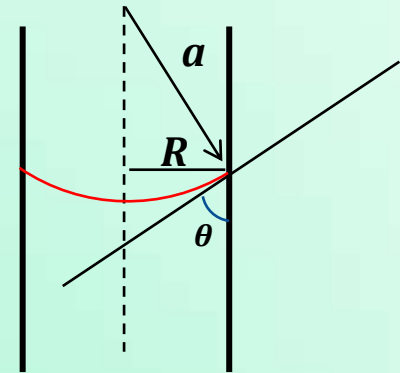


# Capillary Rise or Depression

$$\cos \theta = \frac{R}{a}$$

Radius of the meniscus:

$$a = \frac{R}{\cos \theta}$$



$$P_1 - P_2 = \frac{2\sigma}{a} = \frac{2\sigma \cos \theta}{R} = (\rho_l - \rho_v)gh$$

Height of the capillary rise:

$$h = \frac{2\sigma \cos \theta}{(\rho_l - \rho_v)gR}$$

# Capillary Rise or Depression

$$h = \frac{2\sigma \cos \theta}{(\rho_l - \rho_v)gR}$$

- It suggests that every point on the meniscus is at the same height  $h$  from the surface of the liquid reservoir, or in other words, the meniscus is flat!
- A more accurate derivation should consider the deviation of meniscus spherical shape in view of the elevation of each point above the flat surface of the liquid. This involves the solution of the general Young-Laplace equation using the expressions for  $R_1$  and  $R_2$ .

Characteristic length scale:

$$L_c = \sqrt{\frac{\sigma}{(\rho_l - \rho_v)g}}$$

# Capillary Rise or Depression

Compare a capillary rise with the tube diameters of 5 mm, 1 mm (laboratory test tube) and 200 nm (capillary diameter of a Redwood tree) in pure water at 20°C. The contact angle of the interface with the tube wall is 20°.

$$\sigma = 0.0728 \text{ N/m}$$

$$h = \frac{2\sigma \cos \theta}{(\rho_l - \rho_v)gR}$$

$$d = 5 \text{ mm}, h = 5.6 \text{ mm}, P_{\text{water}} = 101.3 \text{ kPa}$$

$$d = 1 \text{ mm}, h = 0.03 \text{ m}, P_{\text{water}} = 101 \text{ kPa}$$

$$d = 200 \text{ nm}, h = 140 \text{ m}, P_{\text{water}} = -1.3 \text{ MPa}$$

***Negative Pressure?***

# **Multiphase Flow and Heat Transfer**

ME546

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# Wettability

Usually, a liquid-vapor phase change is accomplished by transferring energy through the walls of a container or channel into or out of a two-phase system.

The vaporization or condensation process ultimately takes place at the liquid-vapor interface.

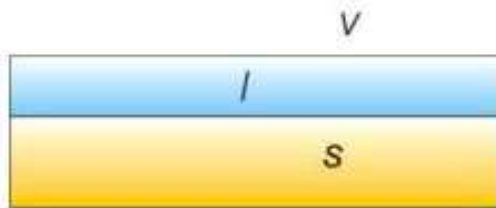
However, the contact through which the energy is transferred will strongly affect the resulting heat and mass transfer in the system.

The performance of heat transfer equipment in which vaporization or condensation occurs may depend strongly on the way that the two phases contact the solid walls.

# Wettability

Liquids with weak affinities for a solid wall will collect themselves into beads while those with high affinities for solid will form film to maximize the liquid-solid contact area.

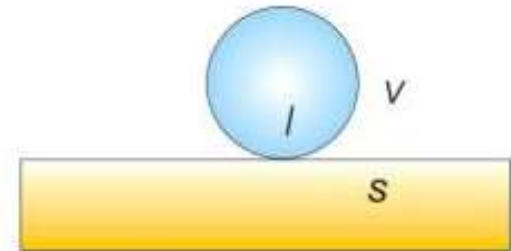
The affinity of liquids for solids  
- wettability of the fluid.



wetting  
 $\Theta = 0^\circ$



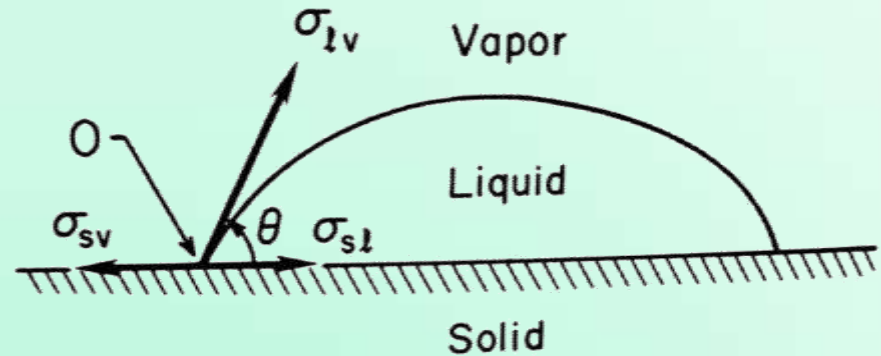
partial wetting  
 $0^\circ < \Theta < 180^\circ$



Nonwetting  
 $\Theta = 180^\circ$

# Neumann's Formula or Young's Equation

$$\sigma_{sv} = \sigma_{sl} + \sigma_{lv} \cos \theta$$



Vertical force  $\sigma_{lv} \sin \theta$  must be balanced by a vertical reaction force in the solid.

Small and also modulus of elasticity of solid is high

No deformation occurs

**$\sigma_{sl}$  and  $\sigma_{sv}$  are not available easily**

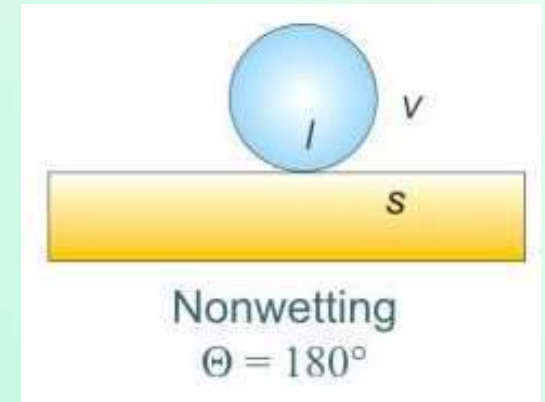
# Neumann's Formula or Young's Equation

As  $\theta \rightarrow 180^\circ$  (if  $g$  is negligible)

Liquid droplet – sphere

one point of contact on solid

Completely non-wetting



As  $\theta \rightarrow 0^\circ$

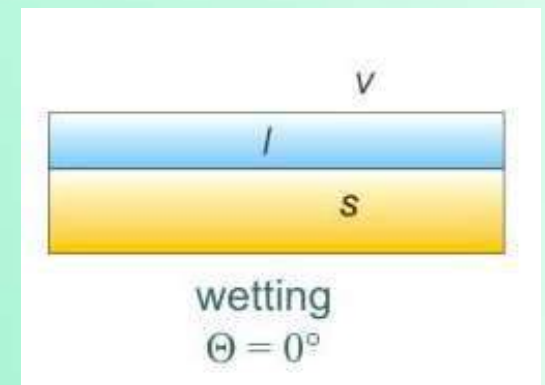
A thin film configuration

Completely wet the solid

Wetting liquid: As  $0^\circ < \theta < 90^\circ$

Non-wetting liquid: As  $90^\circ < \theta < 180^\circ$

one point of contact on solid

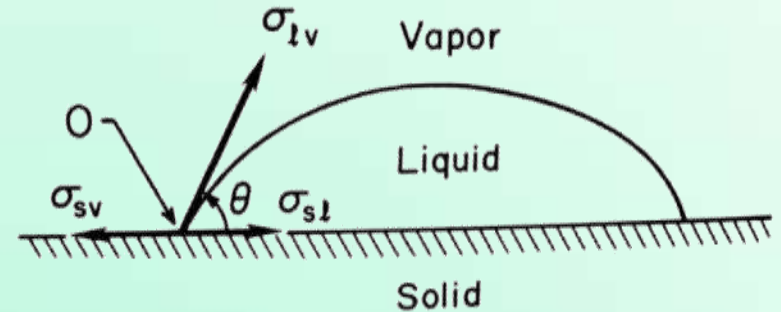


# Neumann's Formula or Young's Equation

$$|\cos \theta| \neq 1$$

At equilibrium:

$$\left| \frac{\sigma_{sv} - \sigma_{sl}}{\sigma_{lv}} \right| < 1$$



If  $\frac{\sigma_{sv} - \sigma_{sl}}{\sigma_{lv}} < -1$   $\sigma_{sl}$  pulls the contact line  $\theta \rightarrow 180^\circ$

Never happens for a droplet surrounded by its vapor, but could happen for a liquid droplet in another immiscible liquid

If  $\frac{\sigma_{sv} - \sigma_{sl}}{\sigma_{lv}} > 1$   $\sigma_{sv}$  pulls the contact line  $\theta \rightarrow 0^\circ$

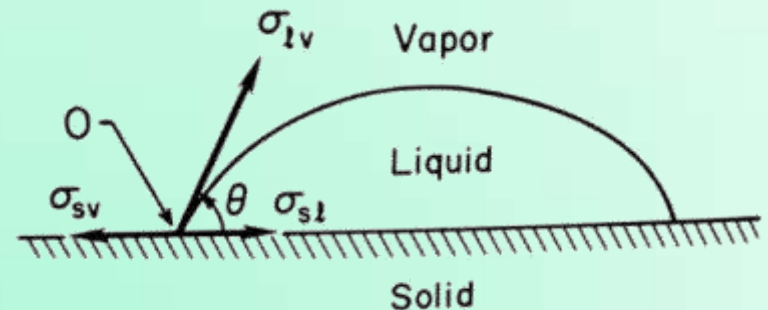
Until the film becomes so thin that molecular interactions come into play.

# Spreading Coefficient

$$S_{ls} = \sigma_{sv} - (\sigma_{lv} + \sigma_{sl})$$

$S_{ls}$  measures the difference between the surface energy  $\sigma_{sv}$  and its value in the case of complete wetting.

$$S_{ls} = \sigma_{lv} (\cos \theta_{ls} - 1)$$



$S_{ls} > 0$ , the liquid will wet the solid and spontaneously spread into a thin film.

$S_{ls} < 0$ , the liquid will partially wet the solid and establish an equilibrium contact angle.

These results are theoretical and no lack of  $\sigma_{sv}$  and  $\sigma_{sl}$ .

# Work of Adhesion

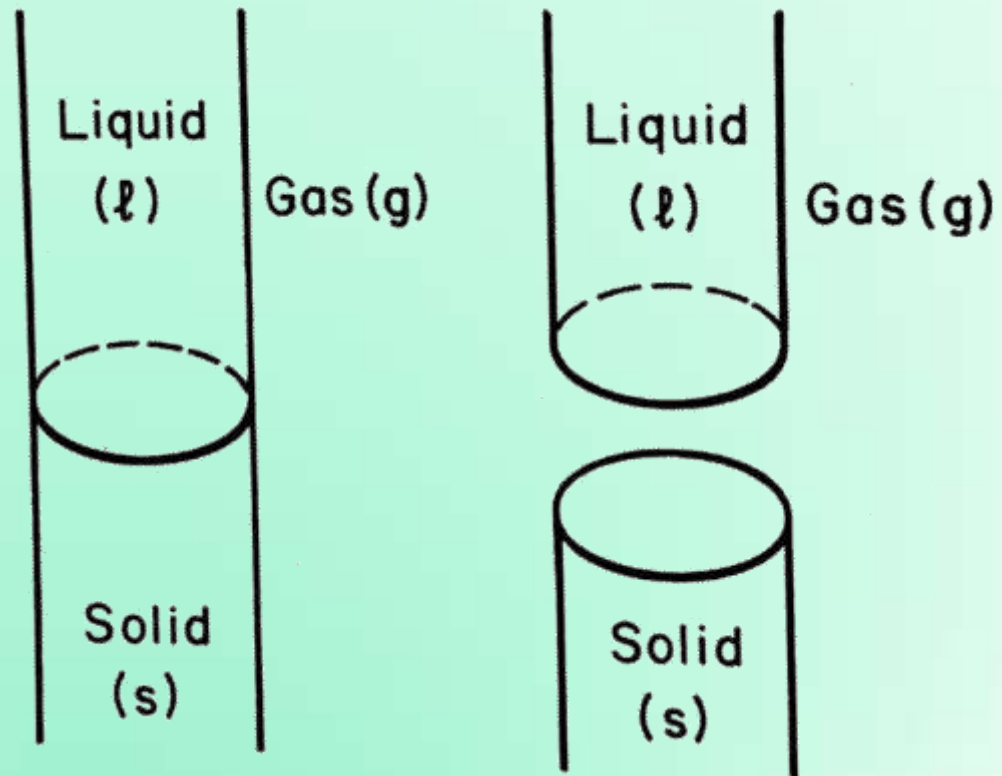
Cylindrical column formed by  $l$ ,  $s$  and low density gas,  $g$ .

The net reversible work required per unit of interface area:

$$w_{sl} = \sigma_{lg} + \sigma_{sg} - \sigma_{sl}$$

Essentially,  
Forming 2 new interfaces  
Breaking on interface

**Adhesion:** due to the minimum reversible work required to rear the liquid off the solid surface.



# Work of Cohesion

Now consider, instead of solid-liquid column there is only one liquid column.

To tear a single liquid column in half:

$$w_u = 2\sigma_{lg}$$

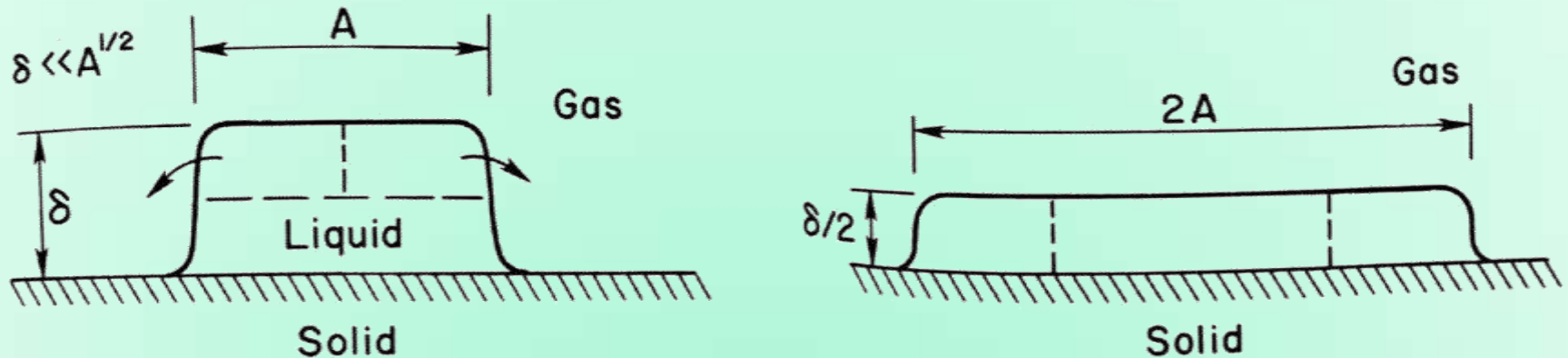
Essentially, two new interfaces are formed without breaking any interface.

**Cohesion:** work required to break internal bonds of the material.

Adhesion – for dissimilar particles  
Cohesion – for similar particles

# Work per Unit Area required for Spreading

Now lets march further. Lets say there is a liquid column of length  $A$ , and height  $\delta$ . We are making it into a liquid column of length  $2A$  and height of  $\delta/2$ .



$$\delta \ll \sqrt{A}$$

$$w_{sp} = \sigma_{lg} + \sigma_{sl} - \sigma_{sg}$$

# Adhesion and Cohesion

$$w_{sp} = \sigma_{lg} + \sigma_{sl} - \sigma_{sg} = -S_{ls}$$

Work interaction is negative ( $S_{ls}$  is positive for spreading)

Work could be extracted if we can ( $= S_{ls}$ )

$$S_{ls} = -w_{sp} = w_{sl} - w_{ll}$$

Spreading coefficient = difference between the work of adhesion and the work of cohesion.

$S_{ls}$  indicates the tendency of the liquid to adhere to the solid relative to its internal cohesive forces.

# Surface Tension

$$w_{sl} = \sigma_{lg} + \sigma_{sg} - \sigma_{sl}$$

Applying it to any two solid or liquid phases  $a$  and  $b$ , and a low density gas or vapor phase  $g$ ,

$$w_{ab} = \sigma_{bg} + \sigma_{ag} - \sigma_{ab}$$

$$\sigma_{ab} = \sigma_{bg} + \sigma_{ag} - w_{ab}$$

The work of adhesion is approximately given as:

$$w_{ab} \cong 2 \left( \sigma_{ag} \sigma_{bg} \right)^{\frac{1}{2}}$$

$$\sigma_{ab} = \sigma_{bg} + \sigma_{ag} - 2 \left( \sigma_{ag} \sigma_{bg} \right)^{\frac{1}{2}}$$

# Surface Tension

For water and hexane in contact with air at 20°C,  $\sigma_{wg}$  is 0.0728 N/m and  $\sigma_{hg}$  is 0.0184 N/m respectively. Use these data to estimate the interfacial tension between hexane and water.

Compare this value to experimentally determined value of  $\sigma_{wh} = 0.0511$  N/m.

# Surface Tension

For water and hexane in contact with air at 20°C,  $\sigma_{wg}$  is 0.0728 N/m and  $\sigma_{hg}$  is 0.0184 N/m respectively. Use these data to estimate the interfacial tension between hexane and water.

Compare this value to experimentally determined value of  $\sigma_{wh} = 0.0511$  N/m.

$$\sigma_{wh} = \sigma_{wg} + \sigma_{hg} - 2 \left( \sigma_{wg} \sigma_{hg} \right)^{\frac{1}{2}}$$

# Surface Tension

$$\sigma_{wh} = \sigma_{wg} + \sigma_{hg} - 2 \left( \sigma_{wg} \sigma_{hg} \right)^{\frac{1}{2}}$$

$$\sigma_{wh} = 0.0180 \text{ N/m}$$

$$S_{hw} = \sigma_{wg} - \sigma_{hg} - \sigma_{wh} = 0.033 \text{ N/m}$$

$S > 0$ , hexane would spontaneously spread over the surface of water

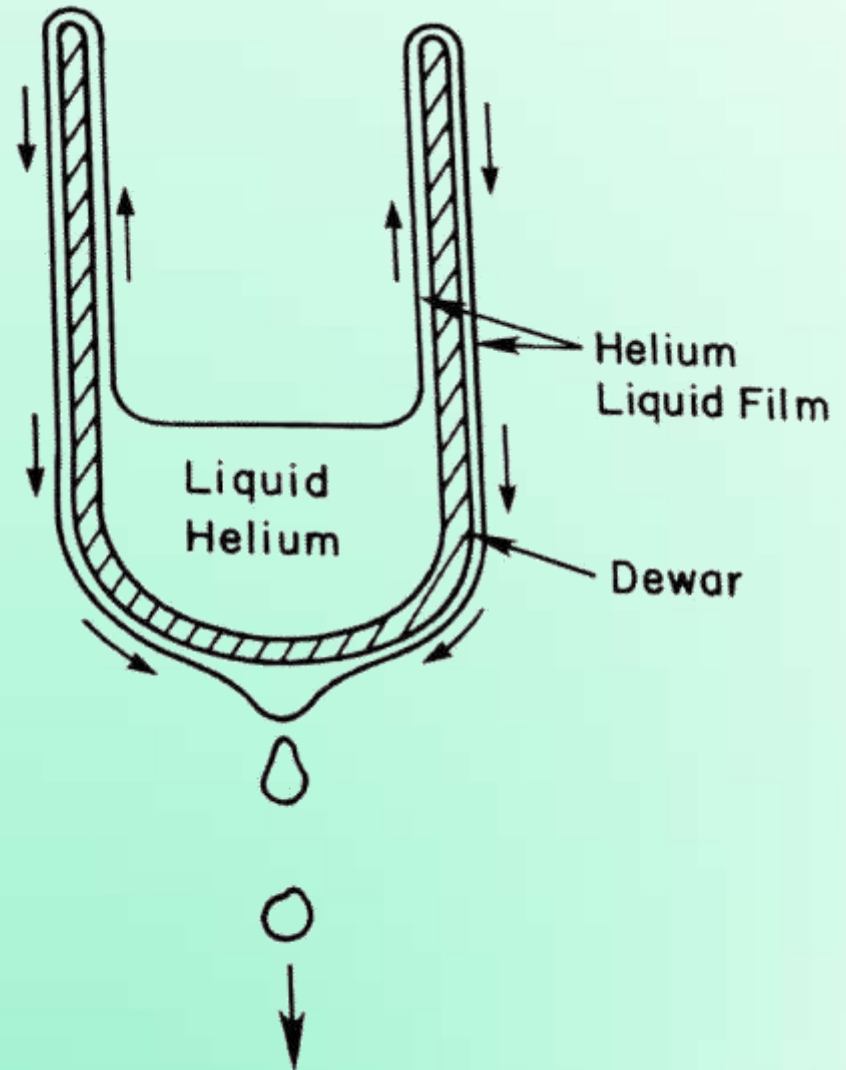
However, close to zero and so tendency is weak.

Hexane is unlikely to form lens-shaped droplets.

Spread out into a film, not aggressively covers the entire liquid surface.

# Spread Thin Films

Spontaneous spreading of liquid helium over the walls of a Dewar flask

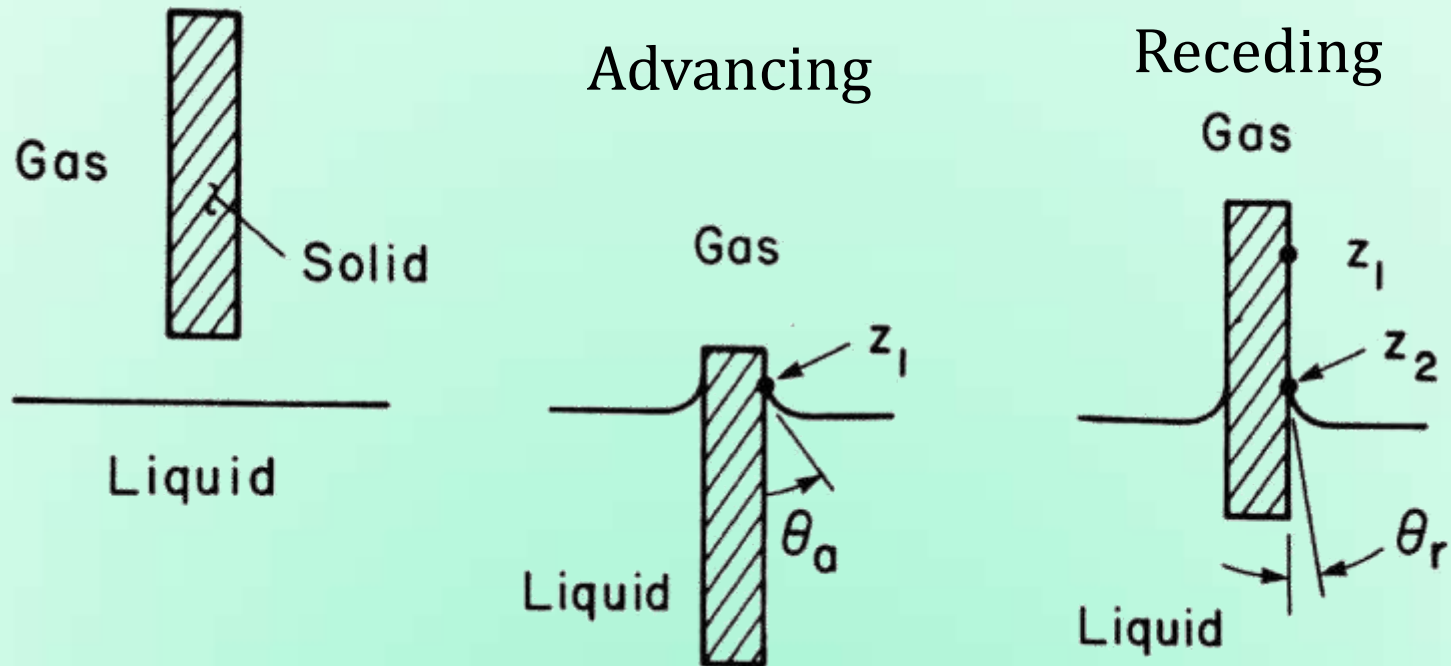


# **Multiphase Flow and Heat Transfer**

ME546

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# Contact Angle Hysteresis, $CAH = \theta_a - \theta_r$



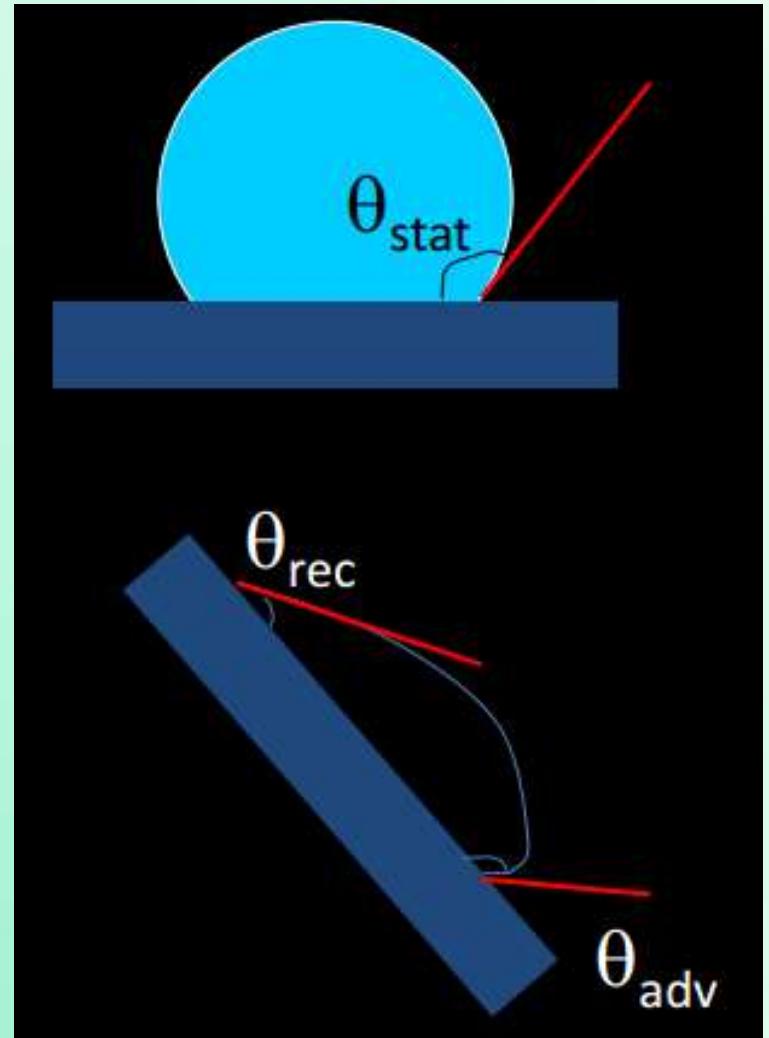
The immersion and removal sequence illustrating advancing and receding contact angles

# Three Different Contact Angles

1. Static contact angle
2. Receding contact angle
3. Advancing contact angle

For a real surface

$$\theta_{rec} < \theta_{stat} < \theta_{adv}$$



# Contact Angle Hysteresis, $CAH = \theta_a - \theta_r$

Generally acknowledged to be a consequence of three factors:

1. Surface inhomogeneity
2. Surface roughness
3. impurities on the surface

For an idealized solid surface that is perfectly smooth, clean and homogeneous in composition,  $\theta_a = \theta_r$

# Applications of Contact Angle Hysteresis

- Coating processes (dynamic hysteresis)
- Digital microfluidics and evaporation of droplets
  - leading to the well-known coffee stain
- Immersion **lithography**
- Fiber coatings
- Ink-jet printing

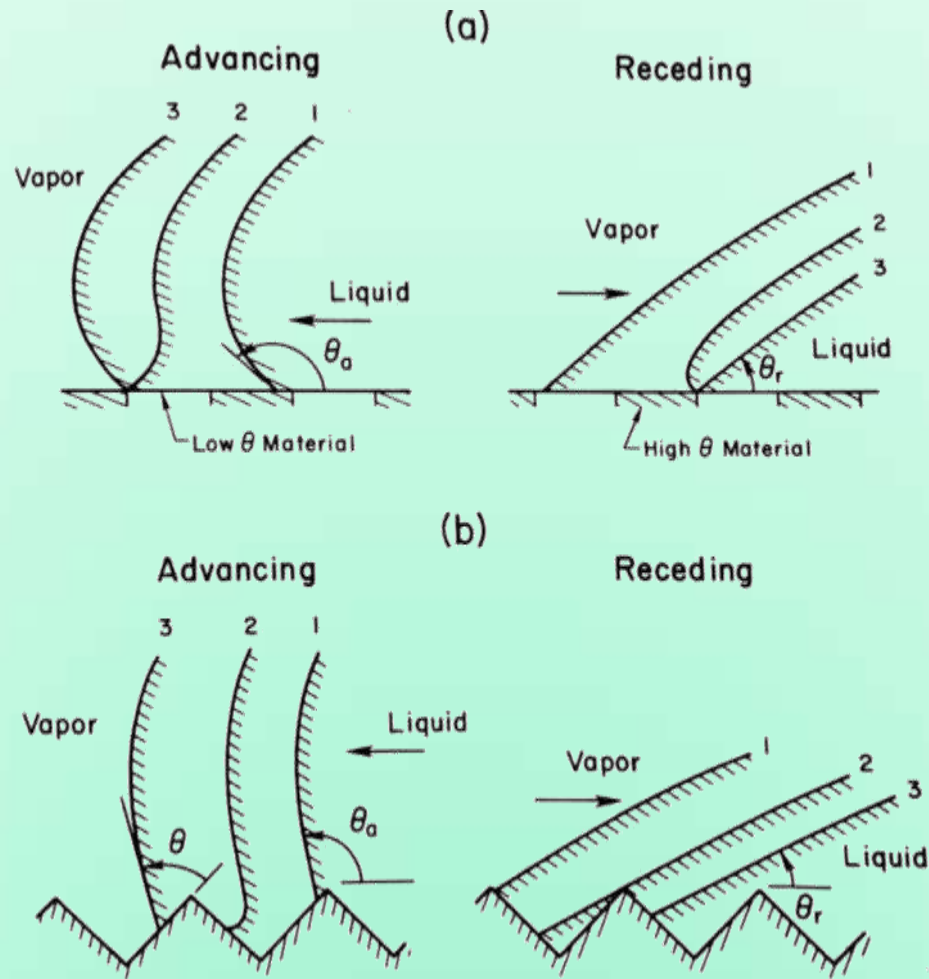
Lithography works on the principle that grease and water repel each other. There is no carving involved. The artist draws on a stone with a greasy crayon (hydrophobic) and then covers the stone with a thin film of water. The oily ink (rolled over the surface) will stick to the greasy image but not to the water-covered areas.

# Applications of Contact Angle Hysteresis

In some cases, hysteresis is a problem (immersion lithography) while in others it is essential (dip-coating).

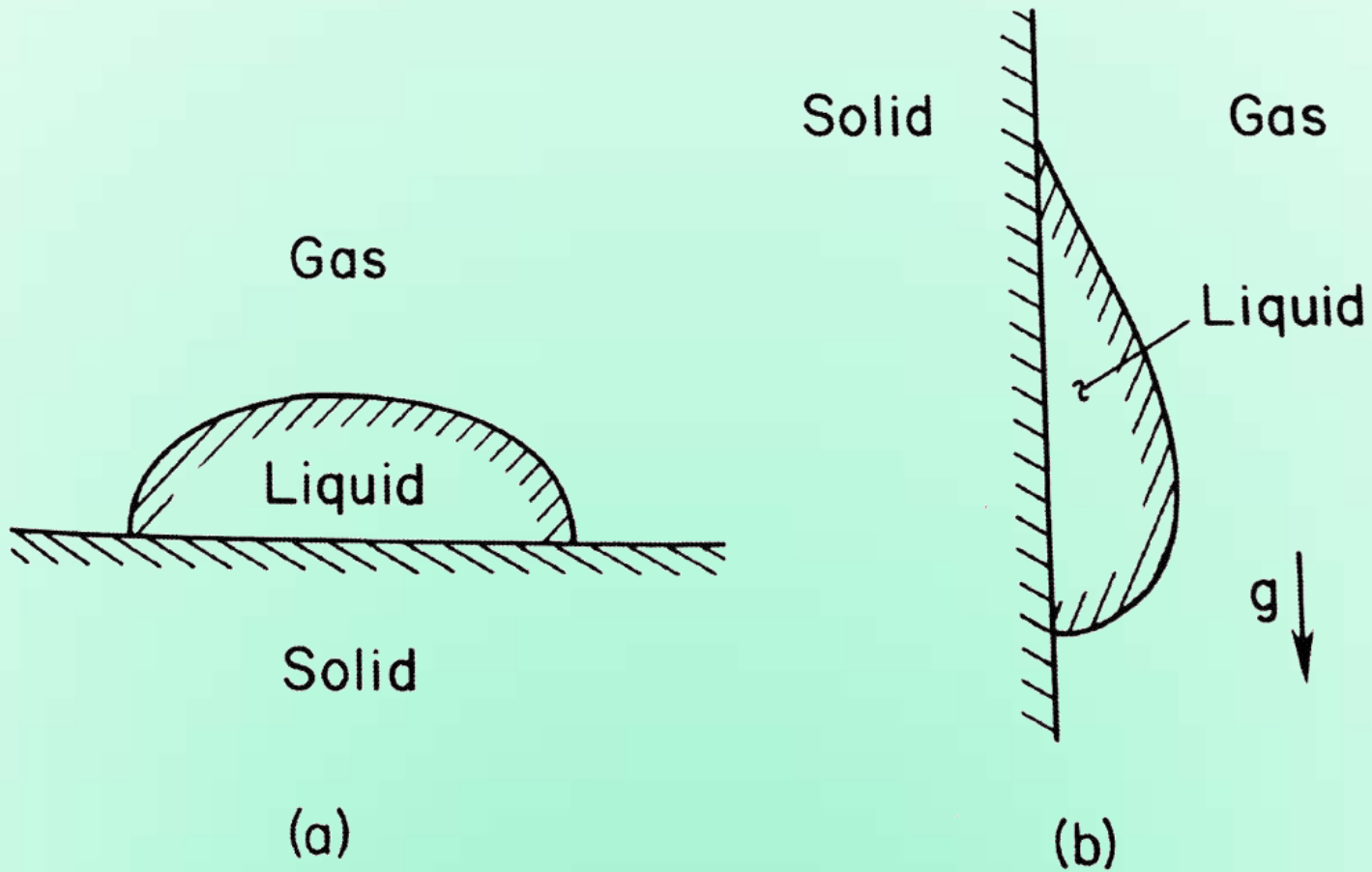
Determining and controlling contact angle hysteresis are critical for the operation of these industrially relevant systems.

# Apparent Contact Angle



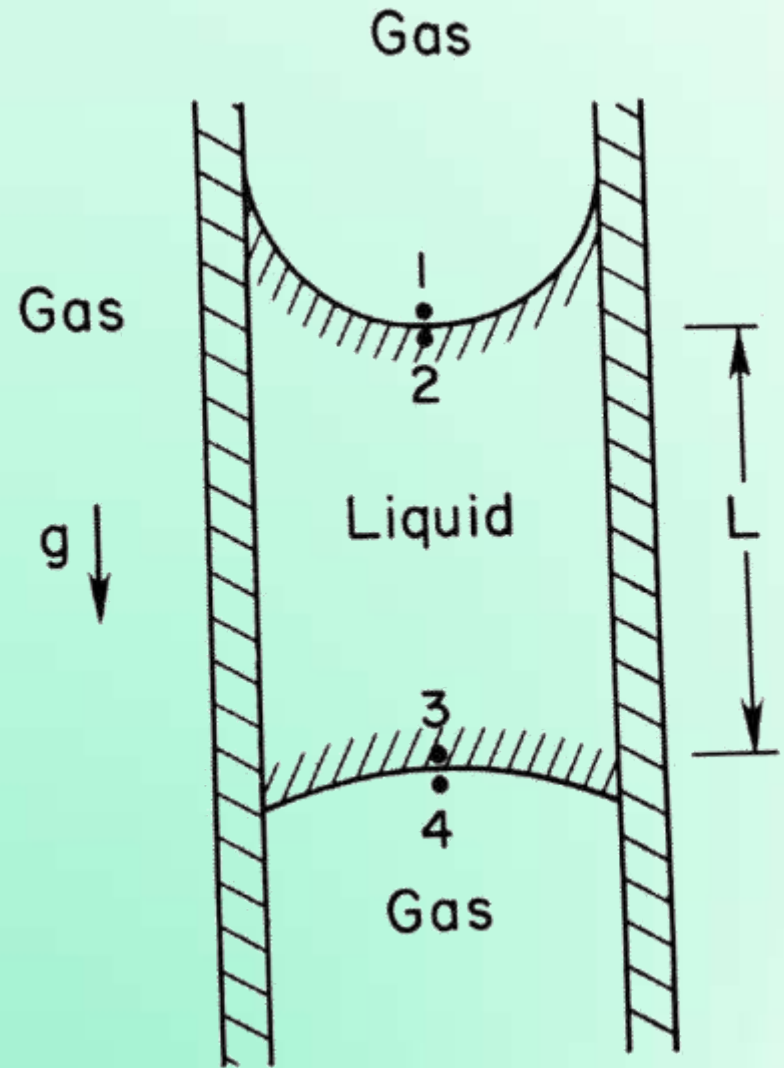
Effects of surface inhomogeneity and roughness on the apparent contact angle for advancing and receding liquid fronts

# Liquid Droplet to Resist Downward Motion



# Liquid Slug

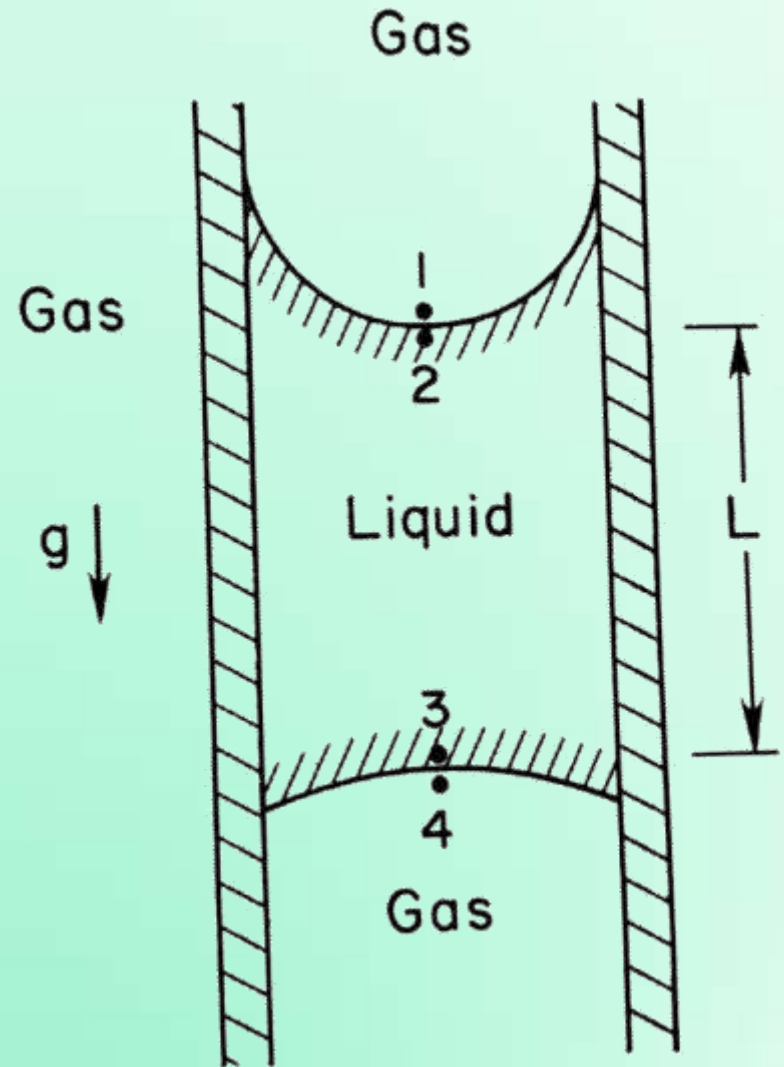
Contact angle hysteresis can allow a liquid slug in a small vertical tube to resist downward motion.



# Liquid Slug

Contact angle hysteresis can allow a liquid slug in a small vertical tube to resist downward motion.

$$P_1 - P_2 = \frac{2\sigma}{r_{top}}$$
$$P_4 - P_3 = \frac{2\sigma}{r_{bottom}}$$



# Liquid Slug

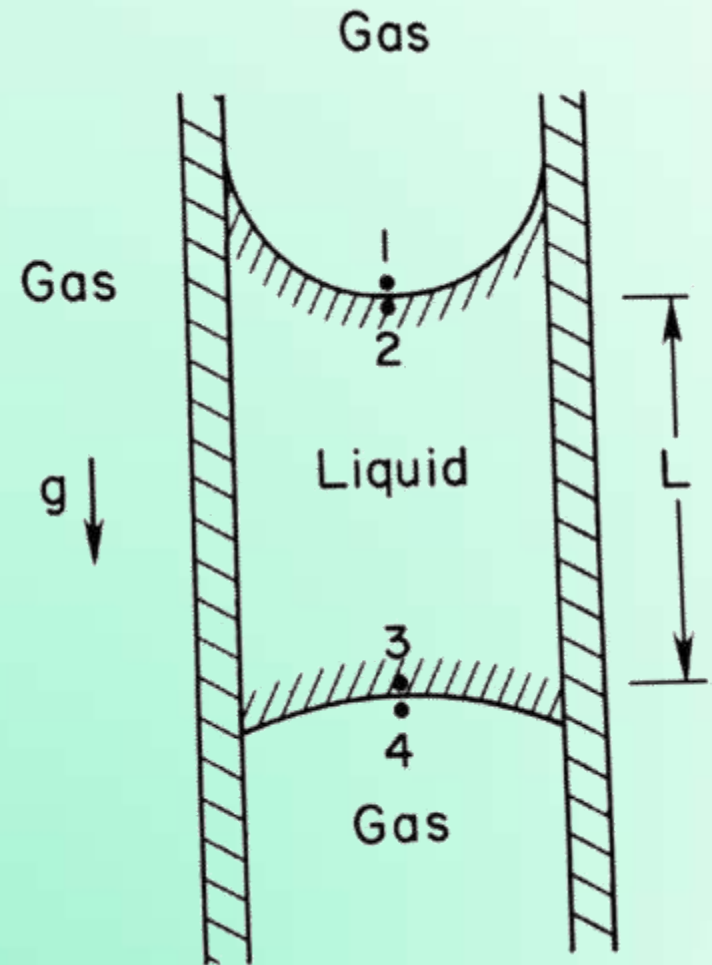
$$P_1 - P_2 = \frac{2\sigma}{r_{top}}$$

$$P_4 - P_3 = \frac{2\sigma}{r_{bottom}}$$

$$P_4 - P_1 = \rho_g gL$$

$$P_3 - P_2 = \rho_l gL$$

$$2\sigma \left( \frac{1}{r_{top}} - \frac{1}{r_{bottom}} \right) = (\rho_l - \rho_g)gL$$



The length of a slug depends on contact angle hysteresis

# Coffee Stain Effect

# Coffee Stain Effect

- Coffee stain phenomenon - real undesired nuisance and a limiting physical factor in: inkjet printing of circuits, OLED displays, or drying of paint, and so on.
- For optimal efficiency the molecules should be distributed homogeneously, but in practice predominant at rim.

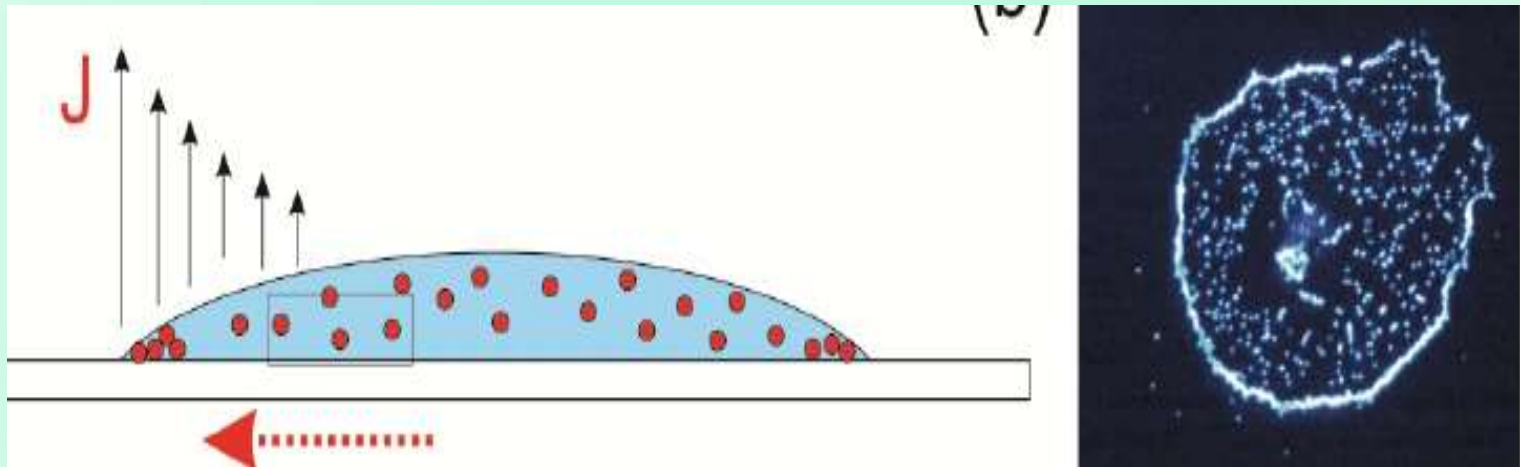
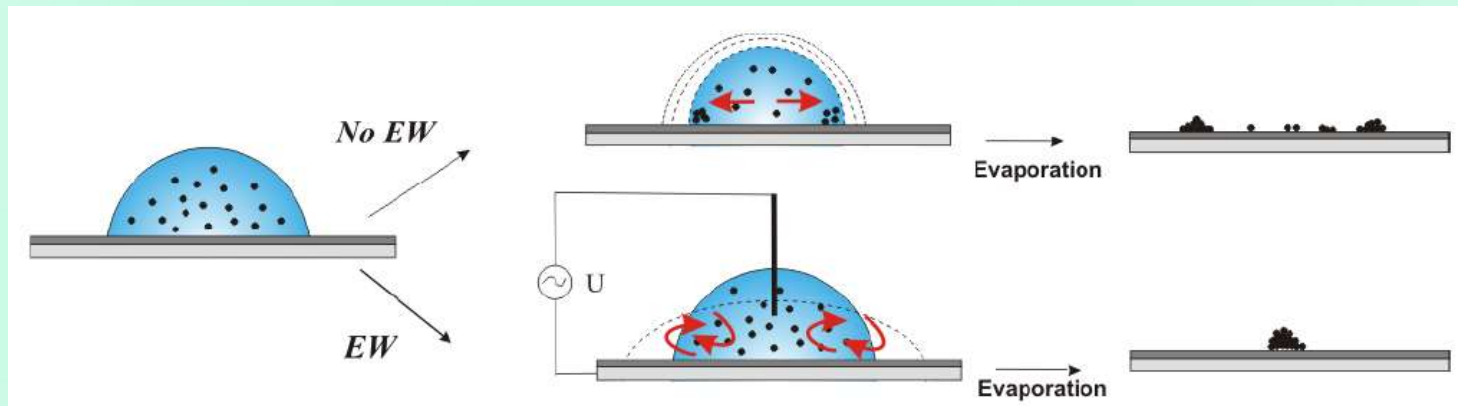


Illustration of coffee stain formation in Panel (a) and a microscope image of a coffee stain formed by fluorescently labeled 5  $\mu\text{m}$  particles in Panel (b)

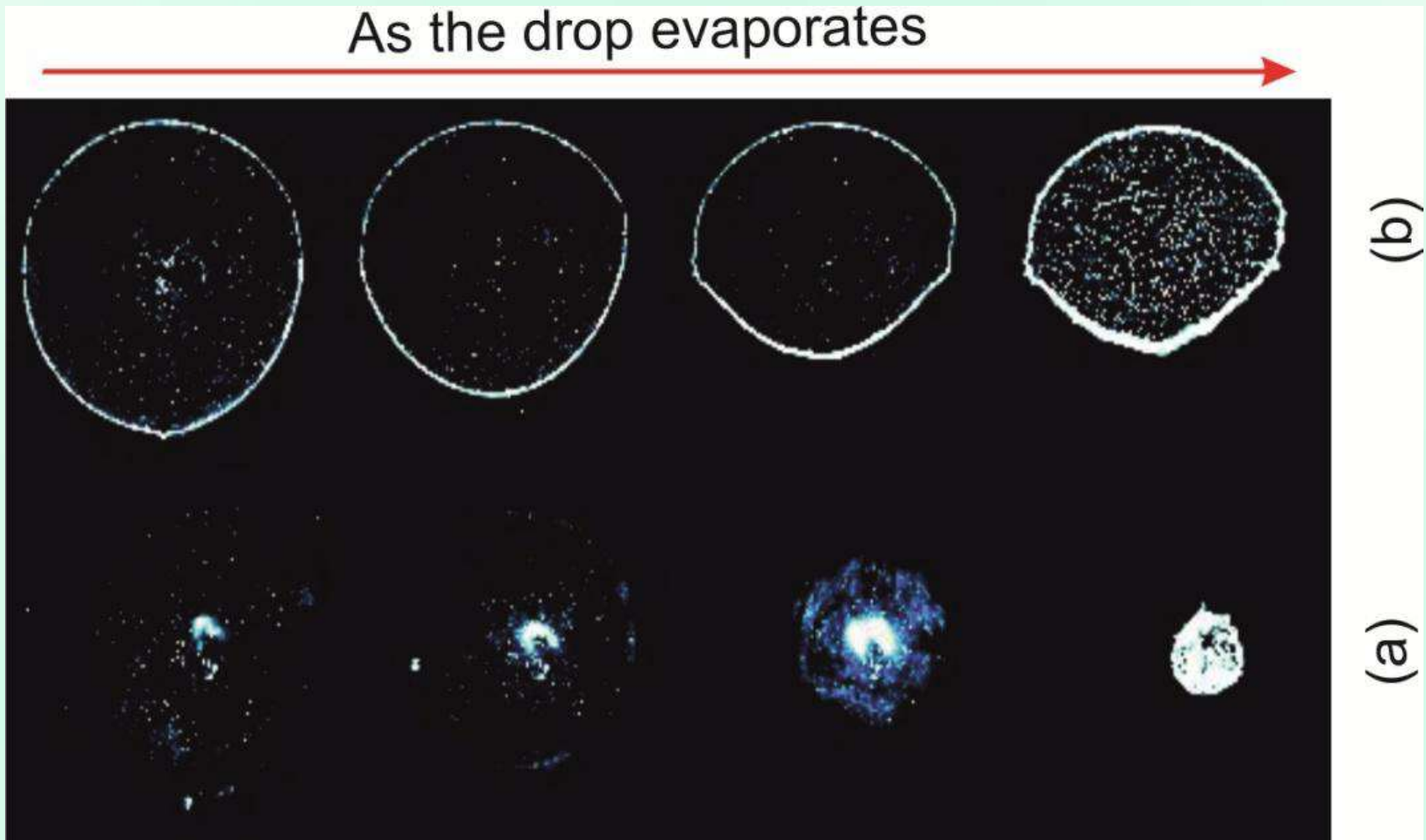
# Coffee Stain Effect

- Suppressed by applying electrowetting (EW) with an alternating voltage.
- Electric force that disturbs the force balance at contact line.
- EW with AC frequency alternatively increases and decreases the apparent contact angle essentially depinning the contact line, *i.e.*, not allowing it to get stuck to intrinsic roughness features on the surface.



Suppression of Coffee stain effect using electro wetting

# Drop Allowed to Evaporate



- (a) Without EW leading to a pronounced coffee stain effect
- (b) Treated with EW – leading to almost homogeneous residue

# Hydrophilic and Hydrophobic Surfaces

Hydrophilic – Water loving!

When the solid has a high affinity for water

$\theta < 90^\circ$ , water spreads

High surface energy *e.g.* glass, metals

Superhydrophilic,  $\theta < 5^\circ$

Hydrophobic – Water repelling!

$\theta > 90^\circ$ , Water does not spread

A spherical cap resting on the substrate with  $\theta$

Low energy *e.g.* teflon

Superhydrophobic,  $\theta > 150^\circ$

# Effect of Surface Roughness

$\theta_E$  = Equilibrium contact angle of an ideal surface

$\theta^*$  = Apparent contact angle of a rough surface

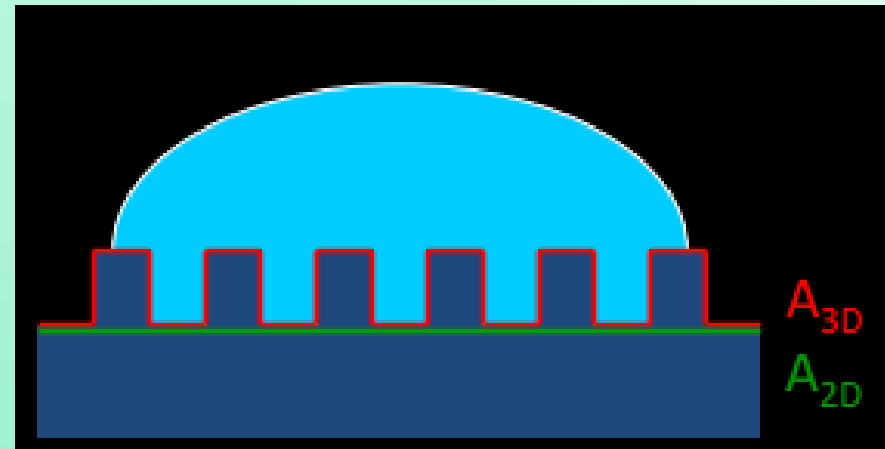
Two models:

- Wenzel state
- Cassie-Baxter state

# Wenzel State

In this state

- There are no air bubbles,
- The droplet is in good contact with the surface.
- Pinned droplet.



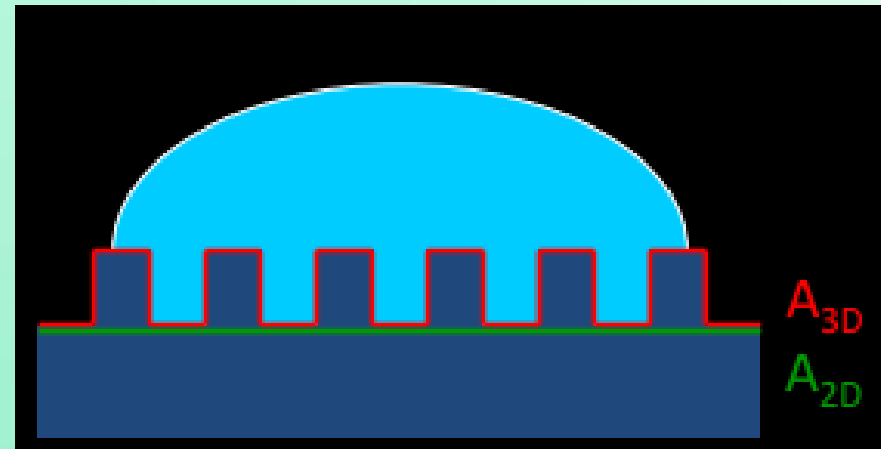
# Wenzel State

In this state

- There are no air bubbles,
- The droplet is in good contact with the surface.
- Pinned droplet.

$$r = \frac{\text{Real surface area}(A_{3D})}{\text{Projected surface area}(A_{2D})}$$

∴ there are no real surfaces,  $r > 1$



# Wenzel State

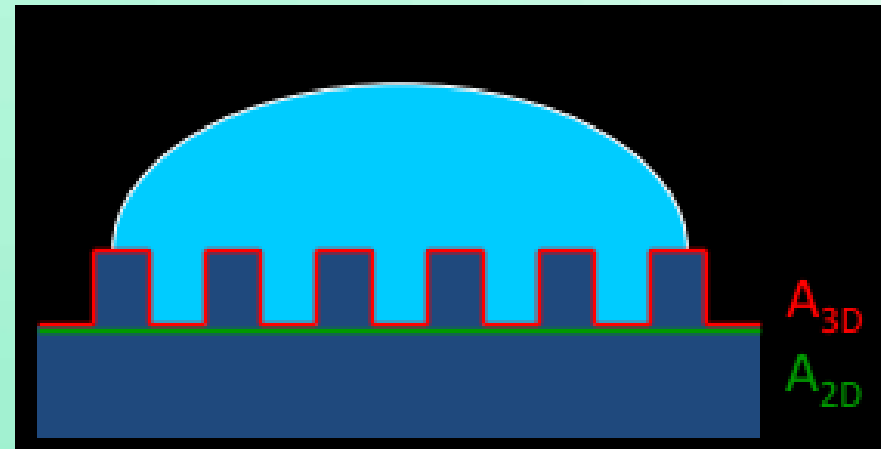
In this state

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- Pinned droplet.

$$r = \frac{\text{Real surface area}(A_{3D})}{\text{Projected surface area}(A_{2D})}$$

∴ there are no real surfaces,  $r > 1$

$$\cos \theta^* = r \cos \theta_E$$



# Consequences of the Wenzel Equation

$$\cos \theta^* = r \cos \theta_E$$

$\therefore$  there are no real surfaces,  $r > 1 \Rightarrow \cos \theta^* > \cos \theta_E$

Hydrophilic:  $\theta_E < 90^\circ$

Say,  $\theta_E = 45^\circ$  and  $r = 1.2$ ,  
Then,  $\theta^* =$

Hydrophobic:  $\theta_E > 90^\circ$

Say,  $\theta_E = 135^\circ$  and  $r = 1.2$ ,  
Then,  $\theta^* =$

# Consequences of the Wenzel Equation

$$\cos \theta^* = r \cos \theta_E$$

$\therefore$  there are no real surfaces,  $r > 1 \Rightarrow \cos \theta^* > \cos \theta_E$

Hydrophilic:  $\theta_E < 90^\circ$

Hydrophobic:  $\theta_E > 90^\circ$

Say,  $\theta_E = 45^\circ$  and  $r = 1.2$ ,  
Then,  $\theta^* = 32^\circ$

Say,  $\theta_E = 135^\circ$  and  $r = 1.2$ ,  
Then,  $\theta^* = 148^\circ$

$$\theta^* < \theta_E$$

$$\theta^* > \theta_E$$

Roughness makes a hydrophilic surface more hydrophilic  
and a hydrophobic surface more hydrophobic.

# Cassie-Baxter State

In this state:

Water sits on top tiny air bubbles

Heterogeneous surface

Adhesive force (water-solid) is extremely low

Water droplets roll-off

Self cleaning

Generally for highly rough surfaces

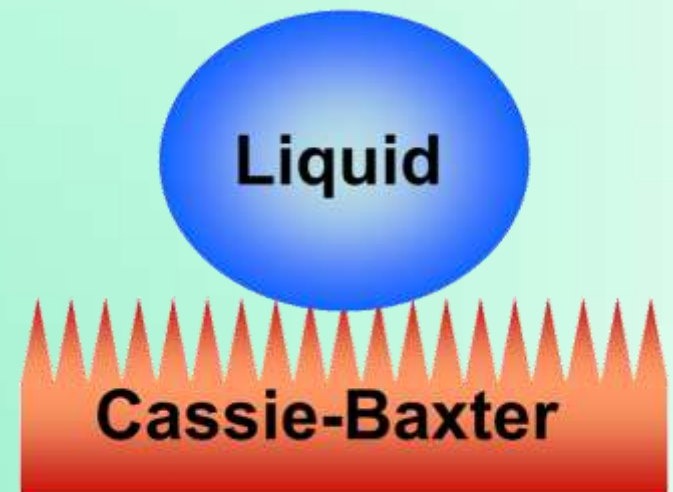


# Cassie-Baxter State

A special case:

$$\cos \theta^* = -1 + \Phi_s (\cos \theta_E + 1)$$

$\Phi_s$  - % of solid in contact with liquid



# Cassie-Baxter State

A special case:

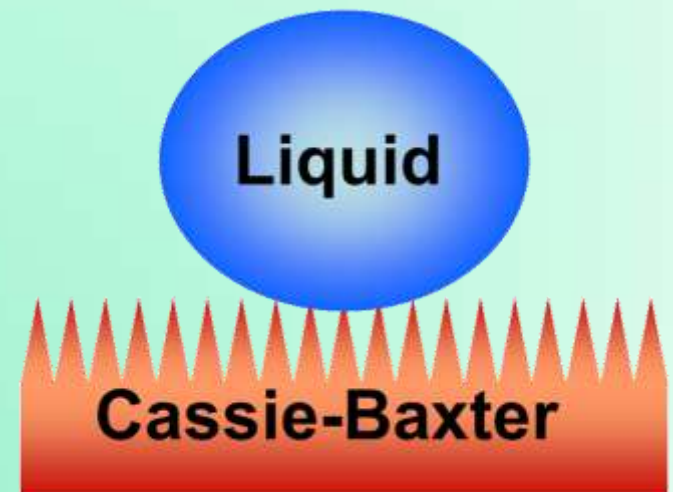
$$\cos \theta^* = -1 + \Phi_s (\cos \theta_E + 1)$$

$\Phi_s$  - % of solid in contact with liquid

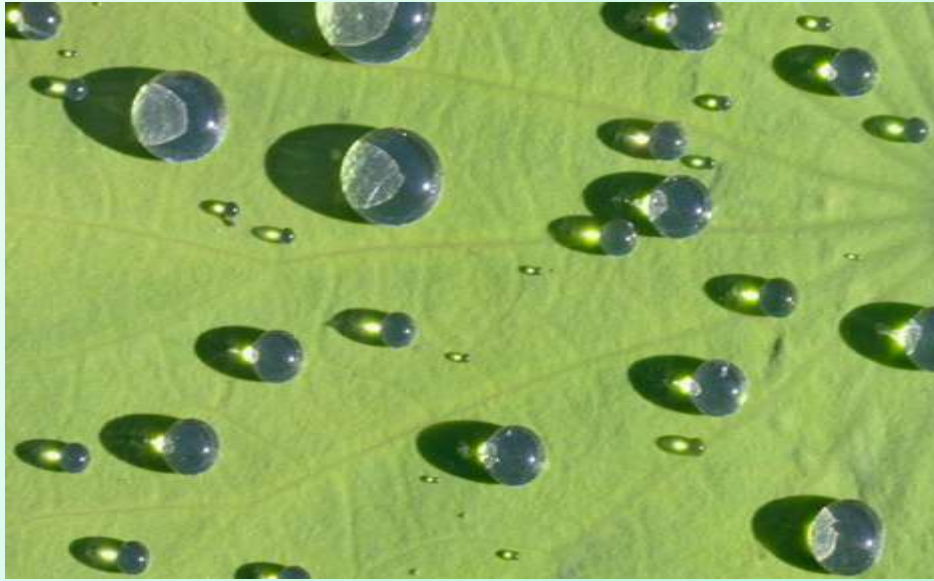
If no solid in contact,  $\Phi_s \rightarrow 0$

$$\cos \theta^* \rightarrow -1$$

$$\theta^* \rightarrow 180^\circ$$



# Petal Effect vs Lotus Effect



Water droplets on Lotus



Water droplets on Rose Petal

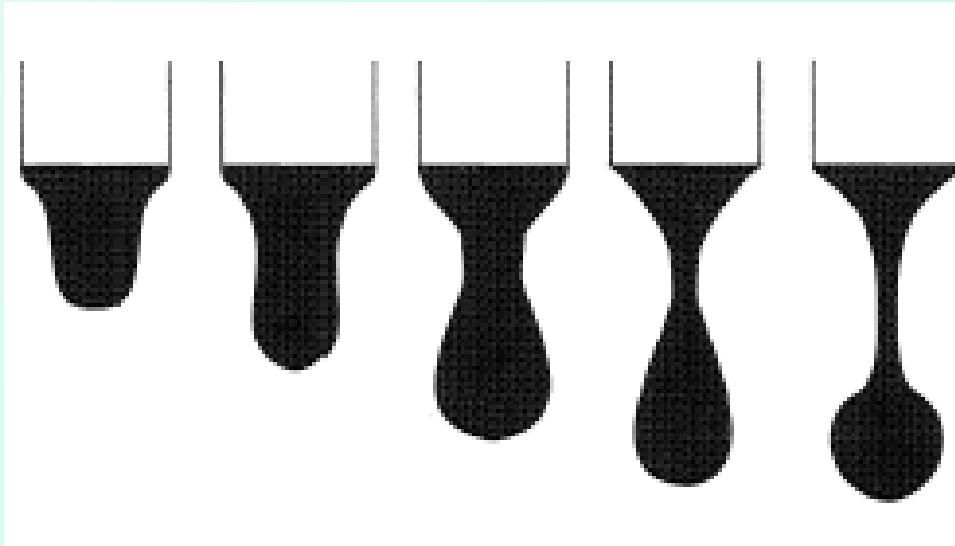


# **Multiphase Flow and Heat Transfer**

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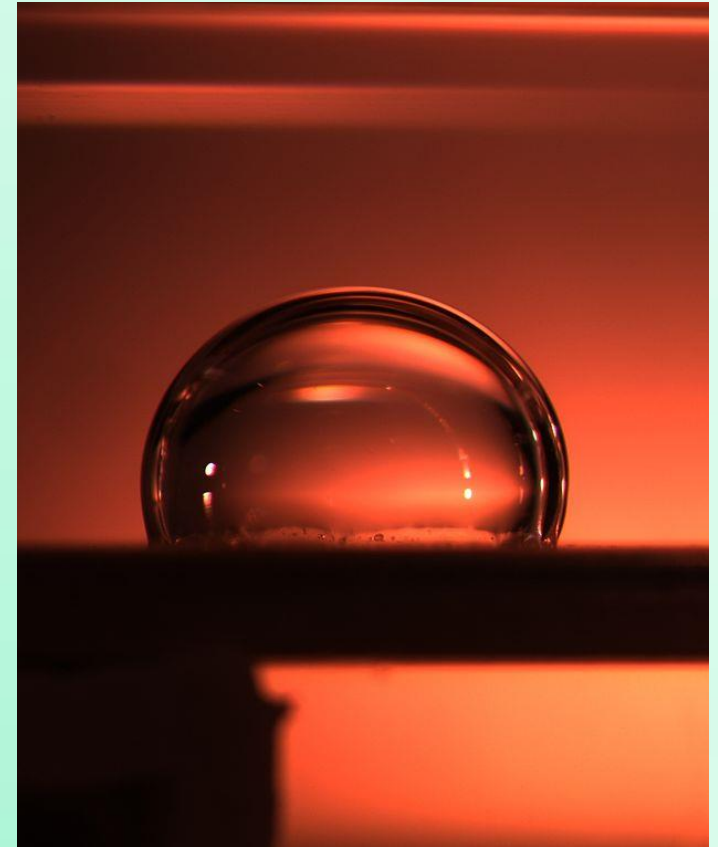
# Pendant and Sessile Drops



Pendant droplet detachment sequence

**Pendant drop:** Drop suspended

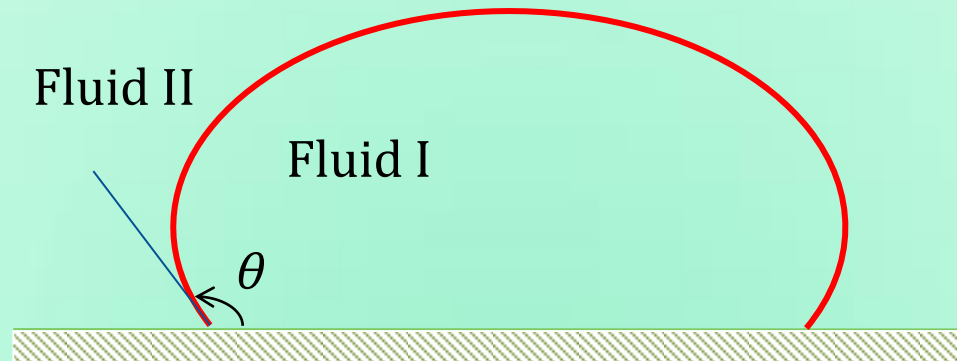
Eg: Droplets formed by the condensation of vapor



Sessile water droplet immersed in oil and resting on a brass surface

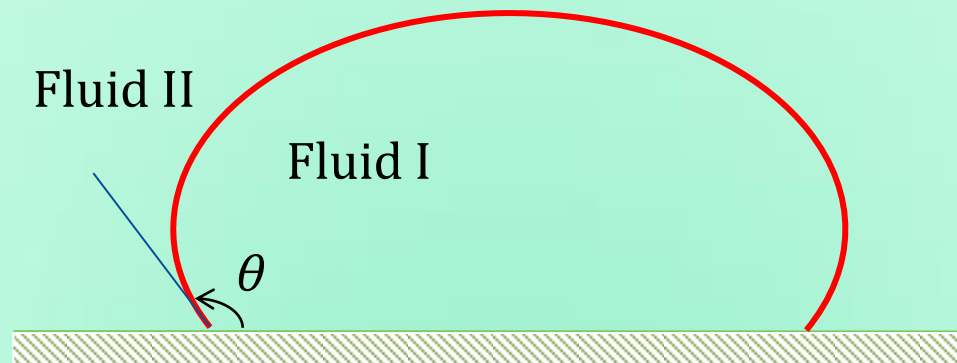
# Interface Shape at Equilibrium

- Consider a sessile drop sitting on a smooth solid surface inside another fluid.



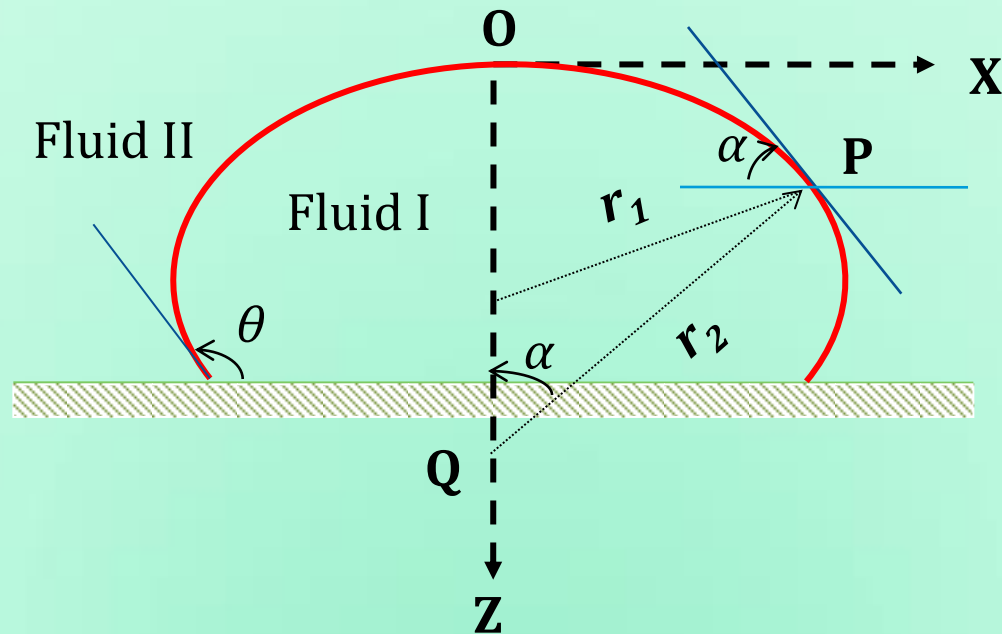
# Interface Shape at Equilibrium

- In the absence of gravity, the drop takes spherical shape with least surface area.
- The drop is deformed by gravity. The center of mass of the drop is forced to be lowered by gravity.
- This increases the surface area, which is opposed by surface tension force.
- Assume there are no external forces acting on the drop.



# Interface Shape at Equilibrium

- Origin of the coordinate system is  $O$ , located at the apex of the surface. The drop is assumed to be axisymmetric.
- At  $O$ : Radii of curvature:  $r_1 = r_2 = r_0$
- At any  $P$ : Radii of curvature are  $r_1$  and  $r_2$

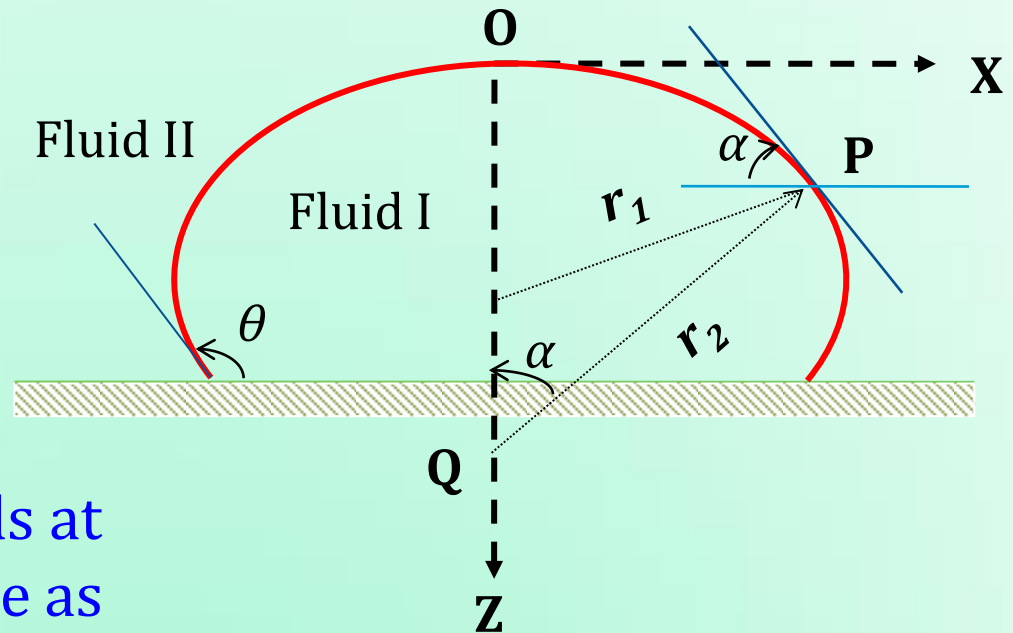


At origin using Young-Laplace:

$$(P_I - P_{II})_0 = \frac{2\sigma}{r_0}$$

At point P:

$$(P_I - P_{II})_p = \sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$$



Hydrostatic pressure heads at any point (P) on interface as seen from fluids I and II:

$$P_{Ip} = P_{I0} + \rho_I g z$$

$$P_{IIp} = P_{II0} + \rho_{II} g z$$

$$r_2 = \frac{x}{\sin \alpha}$$

# Bashforth-Adams Equation

$$\sigma \left( \frac{1}{r_1} + \frac{\sin \alpha}{x} \right) = \frac{2\sigma}{r_0} + (\rho_I - \rho_{II})gz$$

$$\frac{1}{(r_1/r_0)} + \frac{\sin \alpha}{(x/r_0)} = 2 + \left[ \frac{(\rho_I - \rho_{II})gr_0^2}{\sigma} \right] \frac{z}{r_0}$$

## Bashforth-Adams Equation

$$\frac{1}{(r_1/r_0)} + \frac{\sin \alpha}{(x/r_0)} = 2 + \mathbf{Bo} \frac{z}{r_0}$$

$$\mathbf{Bo} = \frac{(\rho_I - \rho_{II})gr_0^2}{\sigma}$$

# Bashforth-Adams Equation

$$\frac{1}{(r_1/r_0)} + \frac{\sin\alpha}{(x/r_0)} = 2 + \mathbf{Bo} \frac{z}{r_0}$$

$$\mathbf{Bo} = \frac{(\rho_I - \rho_{II})gr_0^2}{\sigma}$$

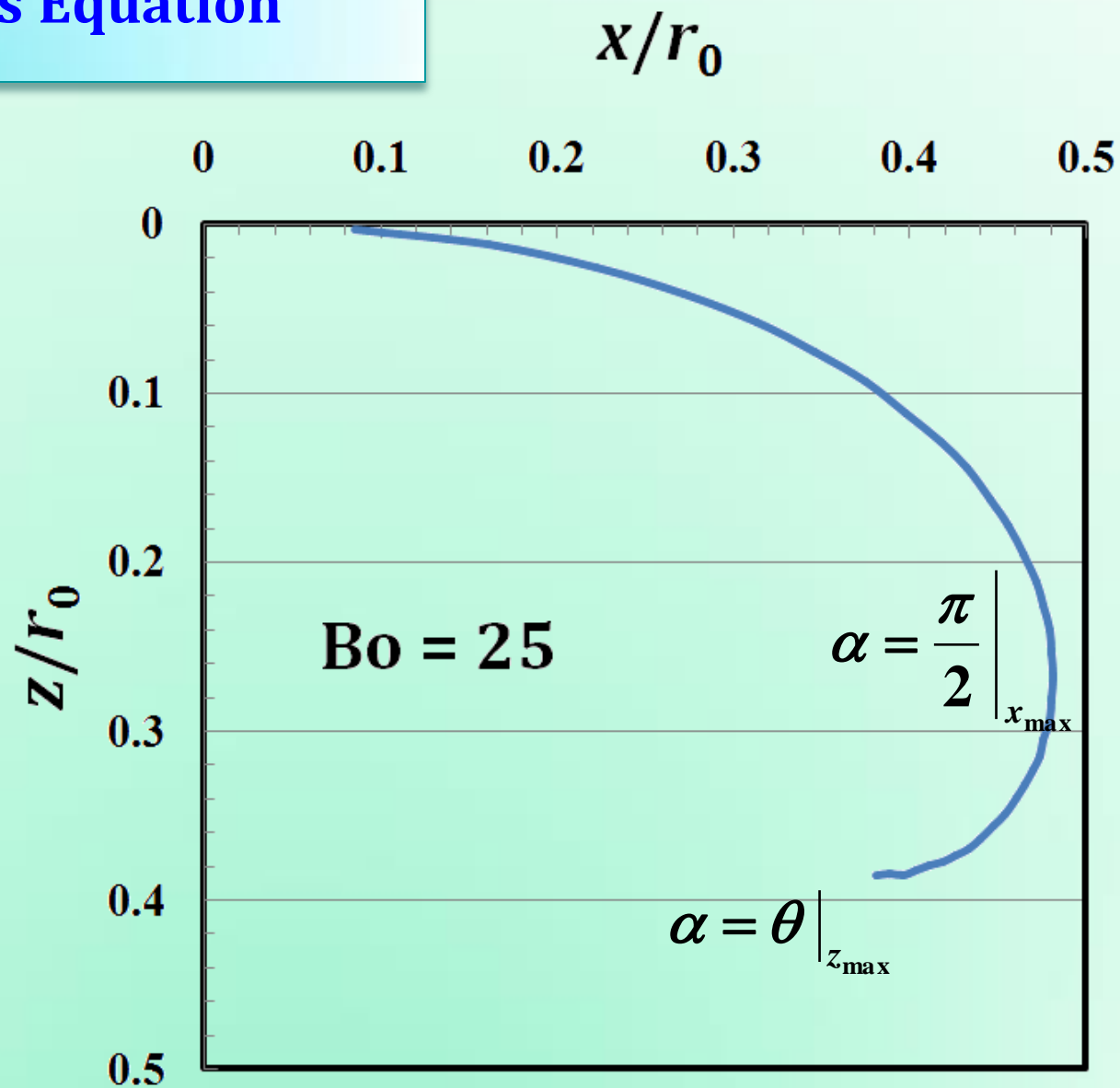
$$r_1 = f(dz/dx, d^2z/dx^2)$$

Numerically solvable with appropriate BCs.

Solution in tabular form (Bashforth and Adams in 1883)

- Variation of  $z/r_0$  and  $x/r_0$  with  $\alpha$  at a given Bo

# Bashforth-Adams Equation



Profile of drop predicted by Bashforth-Adams equation at  $Bo = 25$

# Bond Number

$$\mathbf{Bo} = \frac{(\rho_I - \rho_{II})gr_0^2}{\sigma}$$

Ratio of the gravity force to the force due to surface tension.

$Bo \ll 1$                       Drop will not deform significantly

$Bo \gg 1$                       Large deformation of the drop

$Bo \ll 1$ , if

the drop is small

or the interfacial tension is large

or the density difference between the two liquid is low

$$\mathbf{Bo} = \frac{(\rho_I - \rho_{II})gr_0^2}{\sigma}$$

$$\rho_I > \rho_{II}$$

Bo > 0, drop shape is oblate

Weight of the drop flattens the surface

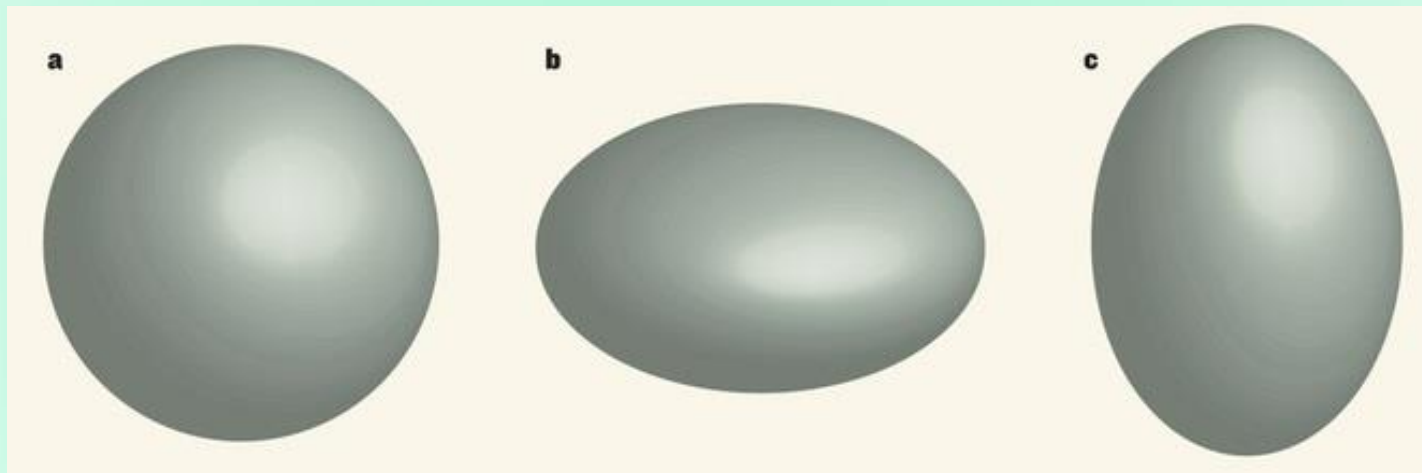
Eg: Rain drop, drop on a surface

$$\rho_I < \rho_{II}$$

Bo < 0, drop shape is prolate

Buoyancy elongates shape vertically

Eg: Vapor bubble in liquid



# Shape of Raindrops



$$\mathbf{Bo} = \frac{(\rho_I - \rho_{II})gr_0^2}{\sigma}$$

$$L_c = \sqrt{\frac{\sigma}{(\rho_l - \rho_v)g}}$$

$$\mathbf{Bo} = \left( \frac{r_0}{L_c} \right)^2$$

$r_0 \ll L_c \Rightarrow$  Gravity is negligible

$L_c \simeq ?$  mm, air-water interface at 25°C

# Shape of Raindrops

$$\mathbf{Bo} = \left( \frac{r_0}{L_c} \right)^2$$

$L_c \simeq 2.7$  mm, air-water interface at 25°C

$r_0 \simeq 2$  mm, Nearly perfect sphere

$r_0 \gg 2$  mm, increasingly flattened

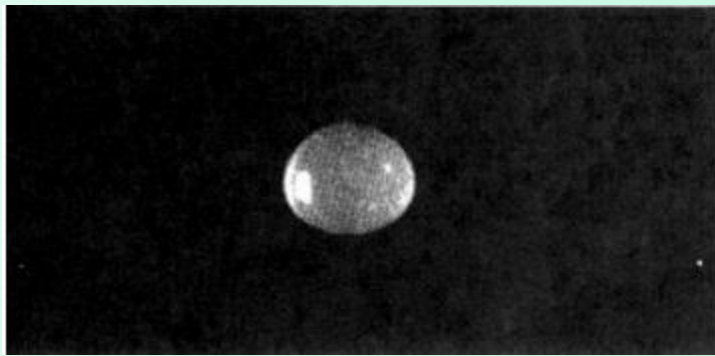


$r_0 > 4.5$  mm, Raindrop breaks into smaller drops due to interaction with air.

Falling raindrops are deformed by the interaction with the air and never take the familiar teardrop shape with a pointed tail and a rounded bottom head.

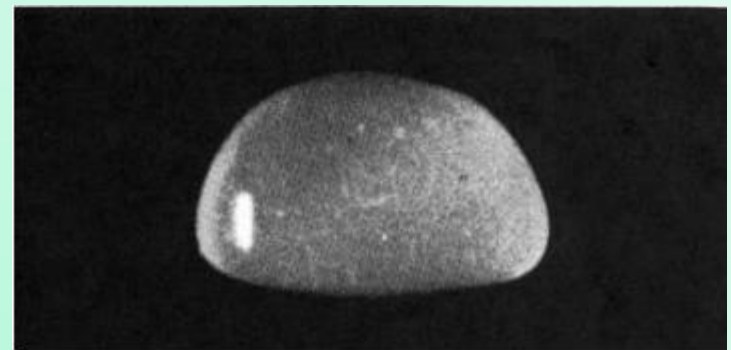
# Shape of Raindrops

Steadily falling raindrops are subject to the combined effects of surface tension, gravity, friction, and air currents.



Small raindrops are dominated by surface tension (nearly spherical)

Here  $r_0$  is slightly greater than  $L_c$  (2.7 mm) and the drop is slightly oval



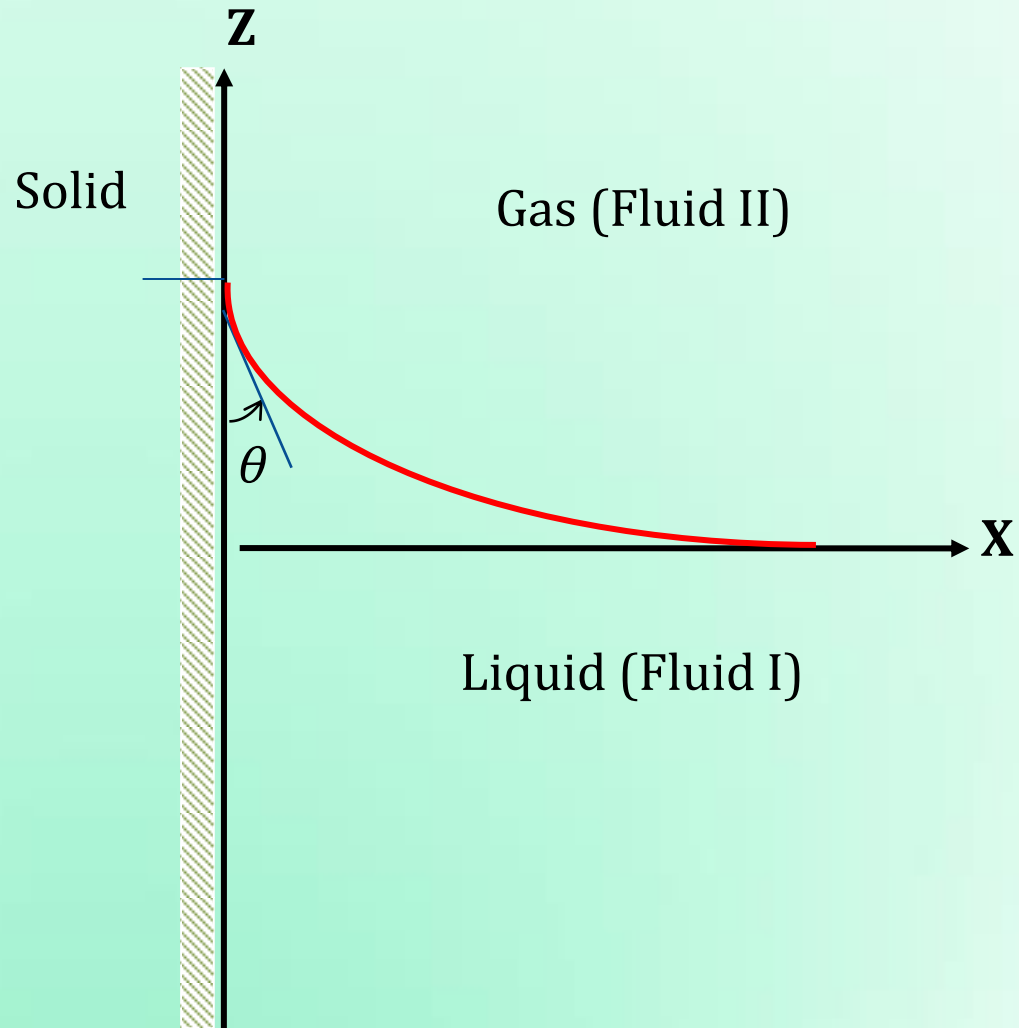
Larger raindrops assume a typical “hamburger” shape

# Liquid Climbing the wall

Shape of a free liquid surface meeting a plane vertical wall.

If the liquid wets the wall ( $\theta < 90^\circ$ ), the liquid level will rise as the wall is approached, meeting the wall at  $\theta$ .

Consider a 2D configuration:



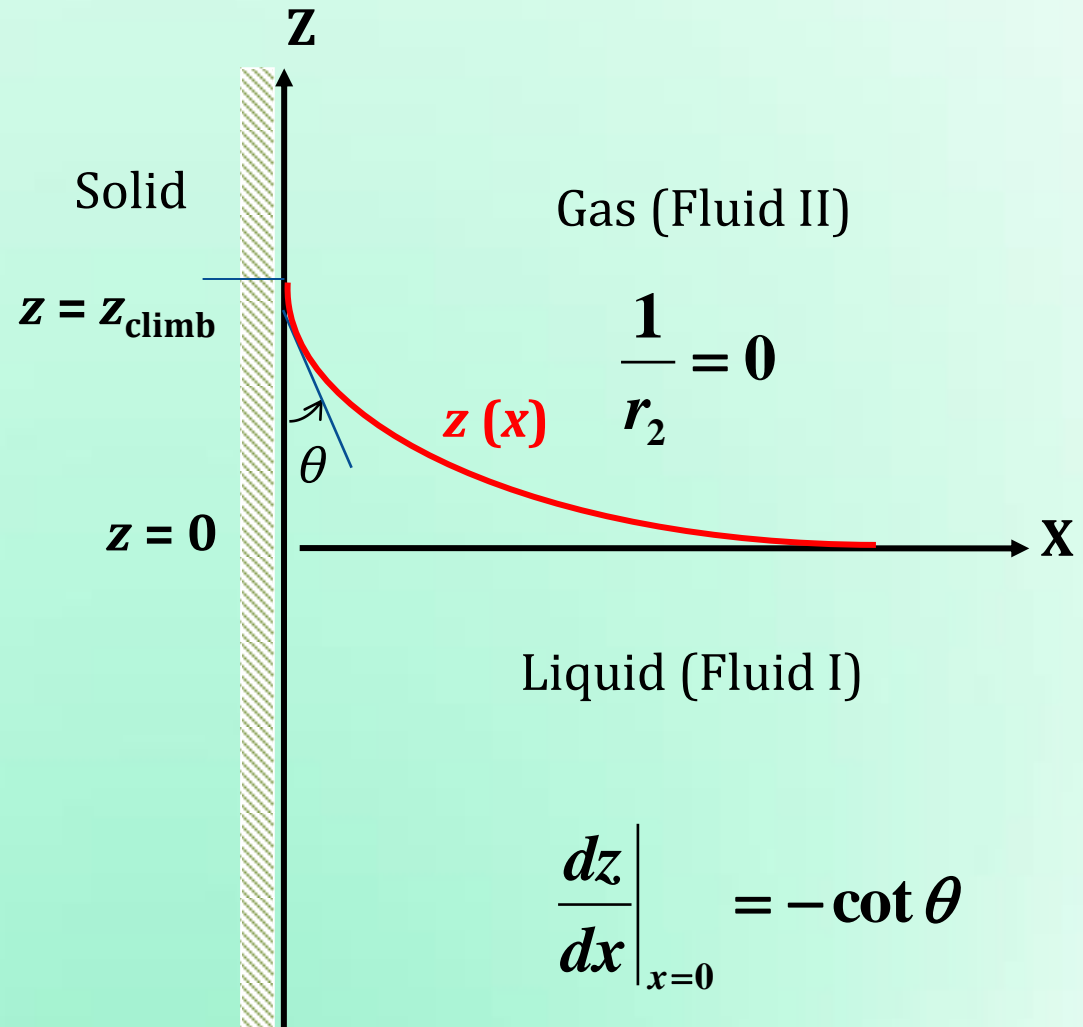
# Liquid Climbing the wall

$$P_I - P_{II} = \sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right)$$

$$P_I = P_0 - \rho_v g z$$

$$P_{II} = P_0 - \rho_l g z$$

$$\frac{(\rho_l - \rho_v) g z}{\sigma} - \frac{1}{r_1} = 0$$



# Radius of Curvature

$$(x - a)^2 + (z - b)^2 = R_c^2$$

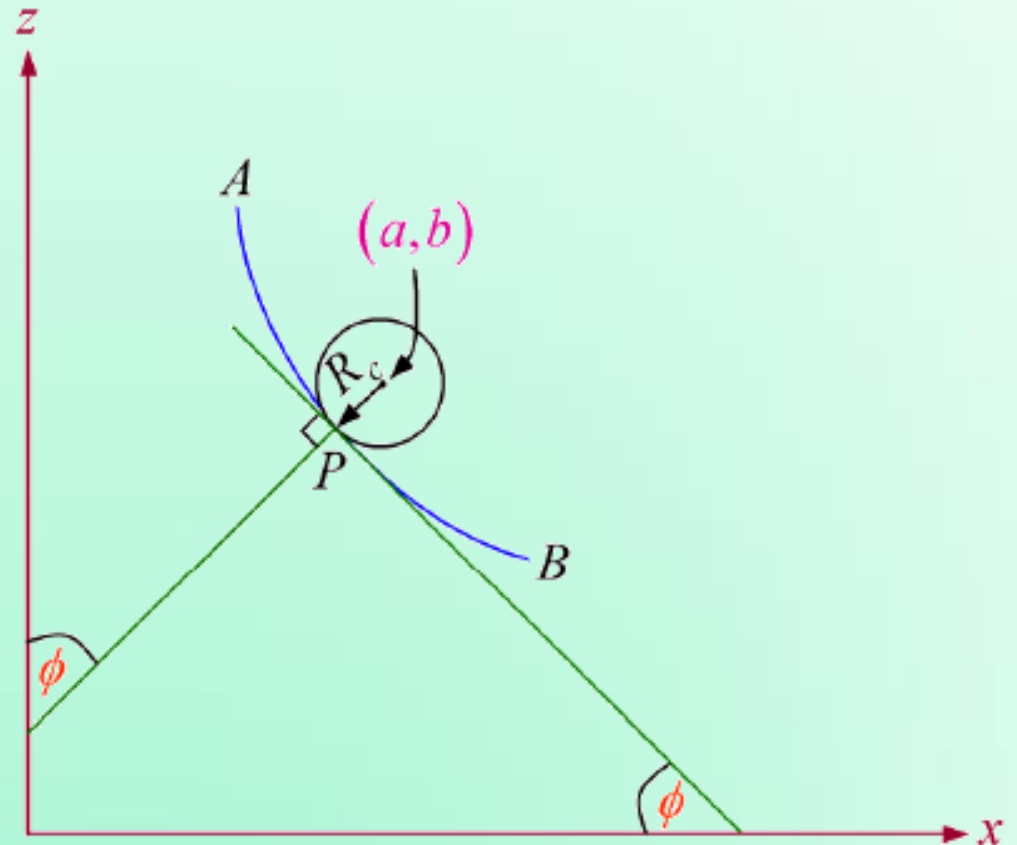
Differentiating w.r.to  $x$

$$\frac{dz}{dx} = -\frac{x - a}{z - b}$$

However

$$\tan \phi = -\frac{dz}{dx}$$

$$\tan \phi = \frac{x - a}{z - b}$$



# Radius of Curvature

$$\cos \phi = \frac{1}{\sqrt{1 + \tan^2 \phi}} = \pm \frac{z - b}{R_c}$$

$$\frac{d \cos \phi}{dz} = \pm \frac{1}{R_c}$$

$$\frac{d \cos \phi}{dz} = \frac{d}{dz} \left[ 1 + \left( \frac{dz}{dx} \right)^2 \right]^{-\frac{1}{2}} = \frac{dx}{dz} \frac{d}{dx} \left[ 1 + \left( \frac{dz}{dx} \right)^2 \right]^{-\frac{1}{2}}$$

$$\frac{1}{R_c} = \pm \frac{d^2 z / dx^2}{\left[ 1 + \left( \frac{dz}{dx} \right)^2 \right]^{\frac{3}{2}}}$$

$$\frac{1}{r_1} = \frac{z''}{\left[1 + (z')^2\right]^{\frac{3}{2}}} \quad z' = \frac{dz}{dx} \quad z'' = \frac{d^2z}{dx^2}$$

$$\frac{(\rho_l - \rho_v)gz}{\sigma} - \frac{1}{r_1} = 0$$

$$\frac{(\rho_l - \rho_v)gz}{\sigma} - \left[1 + (z')^2\right]^{-\frac{3}{2}} z'' = 0$$

Multiplying by  $z'$

$$\frac{(\rho_l - \rho_v)g}{\sigma} z(z') - \frac{1}{2} \left[1 + (z')^2\right]^{-\frac{3}{2}} (2z'z'') = 0$$

## Integrating

$$\frac{(\rho_l - \rho_v)g}{\sigma} \int z(z') - \frac{1}{2} \int [1 + (z')^2]^{-\frac{3}{2}} (2z'z'') = 0$$

Since, 
$$-\frac{1}{2} [1 + (z')^2]^{-\frac{3}{2}} (2z'z'') = \frac{d}{dx} \left\{ [1 + (z')^2]^{-\frac{1}{2}} \right\}$$

$$\frac{(\rho_l - \rho_v)g}{2\sigma} z^2 + [1 + (z')^2]^{-\frac{1}{2}} = C$$

BCs:  $z, z' = 0$  as  $y \rightarrow \infty \Rightarrow$  Integral constant,  $C = 1$

# Liquid Climbing the wall

$$\frac{(\rho_l - \rho_v)g}{2\sigma} z^2 + \left[1 + (z')^2\right]^{-\frac{1}{2}} = 1$$

$$z' \Big|_{x=0} = -\cot \theta$$

The height to which the liquid climbs at the vertical wall

$$z \Big|_{x=0} = z_{\text{climb}} = \left[ \frac{2\sigma(1 - \sin \theta)}{(\rho_l - \rho_v)g} \right]^{\frac{1}{2}}$$

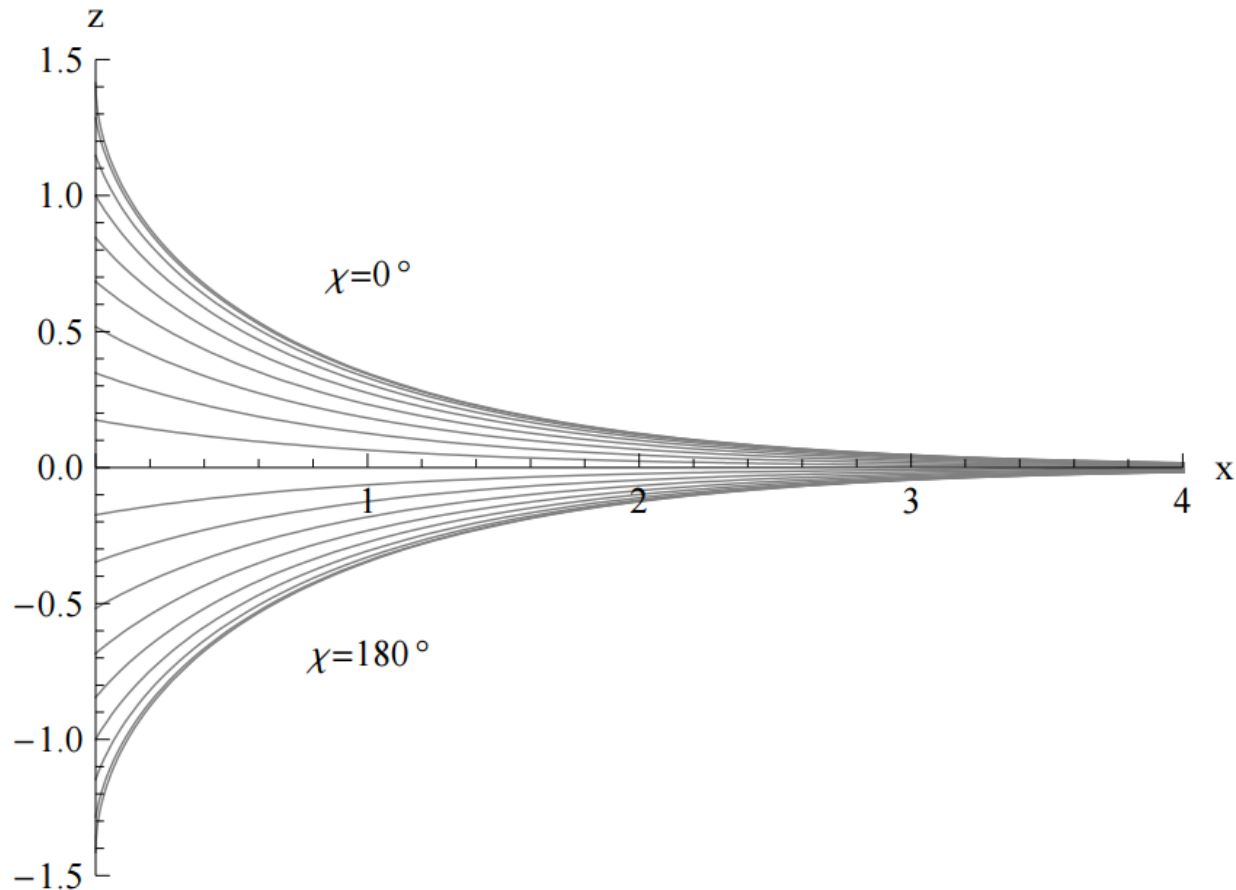
# Liquid Climbing the wall

The shape of the interface:

$$\frac{x}{L_c} = \cosh^{-1}\left(\frac{2L_c}{z}\right) - \cosh^{-1}\left(\frac{2L_c}{z_{\text{climb}}}\right) - \left(4 + \frac{z^2}{L_c^2}\right)^{\frac{1}{2}} + \left(4 + \frac{z_{\text{climb}}^2}{L_c^2}\right)^{\frac{1}{2}}$$

$$L_c = \left[ \frac{\sigma}{(\rho_l - \rho_v)g} \right]^{\frac{1}{2}}$$

# Liquid Climbing the wall



**Figure 5.7.** Flat-wall meniscus (in units of  $L_c = 1$ ) for all contact angles in steps of  $10^\circ$ . Close to the wall the menisci are linear and far from the wall they approach the  $x$ -axis exponentially.

# Capillary Rise or Depression

A cylindrical container is filled with saturated liquid R-134a and its vapor at 32°C. Determine the height to which the liquid will climb the vertical walls of the container if the contact angle with the walls is 5°.

At 32°C,  $\sigma = 0.0072 \text{ N/m}$ ,  $\rho_v = 39.8 \text{ kg/m}^3$ ,  $\rho_l = 1180 \text{ kg/m}^3$ .

$$z_{\text{climb}} = \left[ \frac{2\sigma(1 - \sin \theta)}{(\rho_l - \rho_v)g} \right]^{\frac{1}{2}}$$

$$z_{\text{climb}} = 1.1 \text{ mm}$$

# Marangoni Forces

Any variation in surface tension along an interface will create tangential (shear) forces, known as **Marangoni Forces**.

This variation can arise from inhomogeneous material properties, or from temperature variations.

Unless balanced by other forces, these shear forces cannot be sustained in a liquid at rest - will set it into motion.

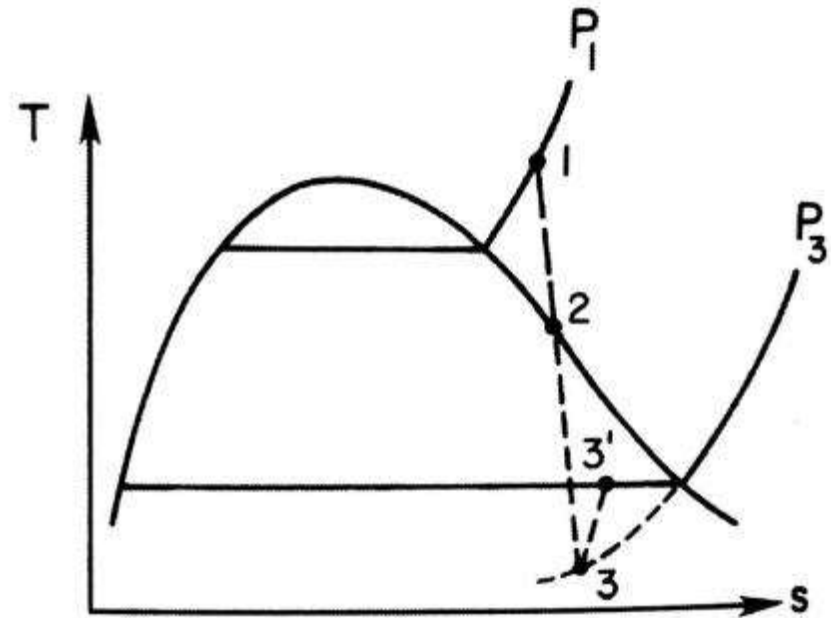
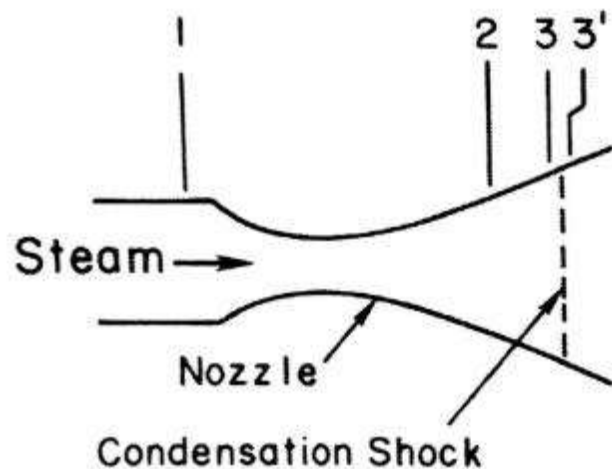
# **Multiphase Flow and Heat Transfer**

ME546

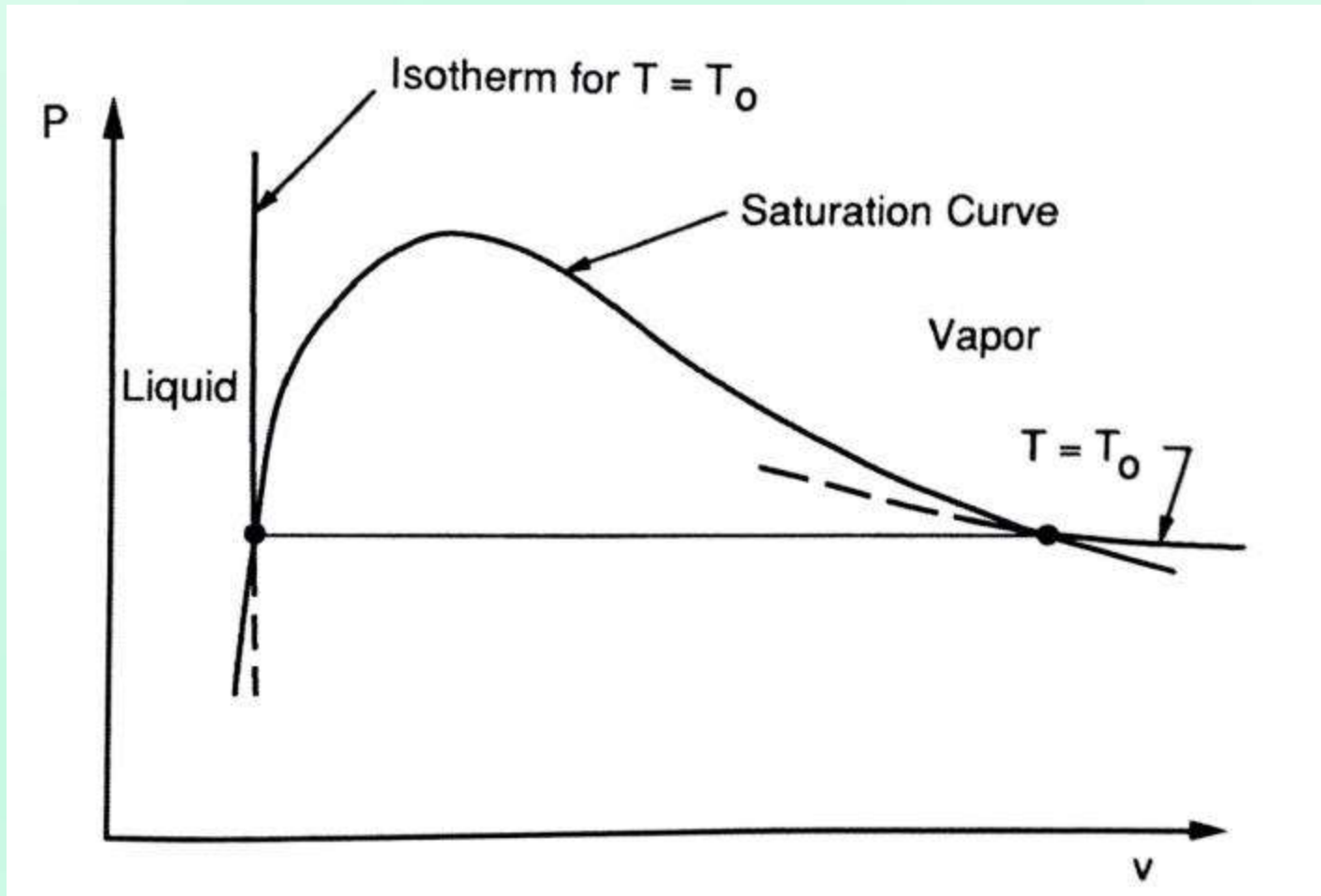
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# Metastable State

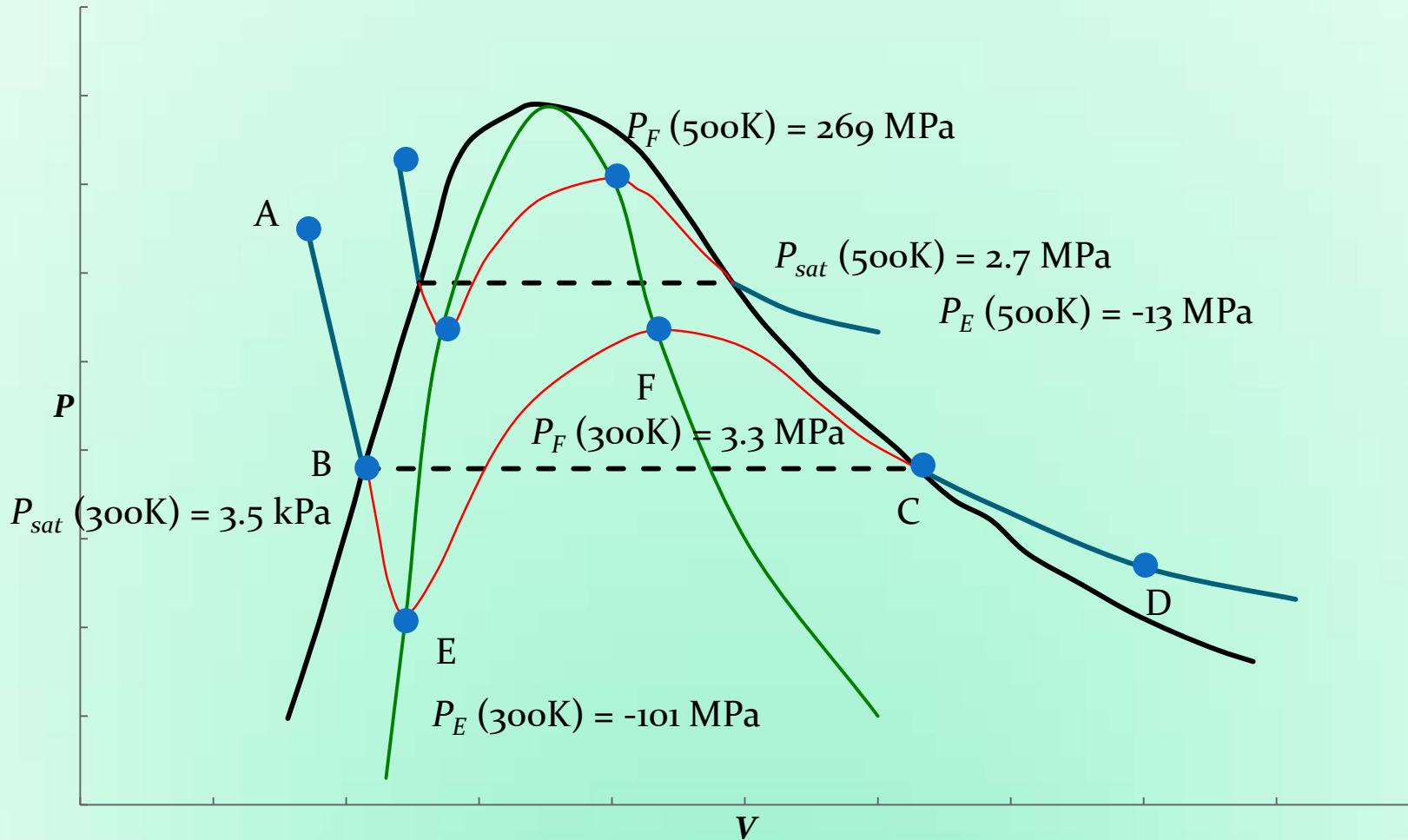
Vapor that is supercooled below its equilibrium saturation temperature and liquid that is superheated above its equilibrium saturation temperature exist in a non-equilibrium condition referred to as a **metastable state**.



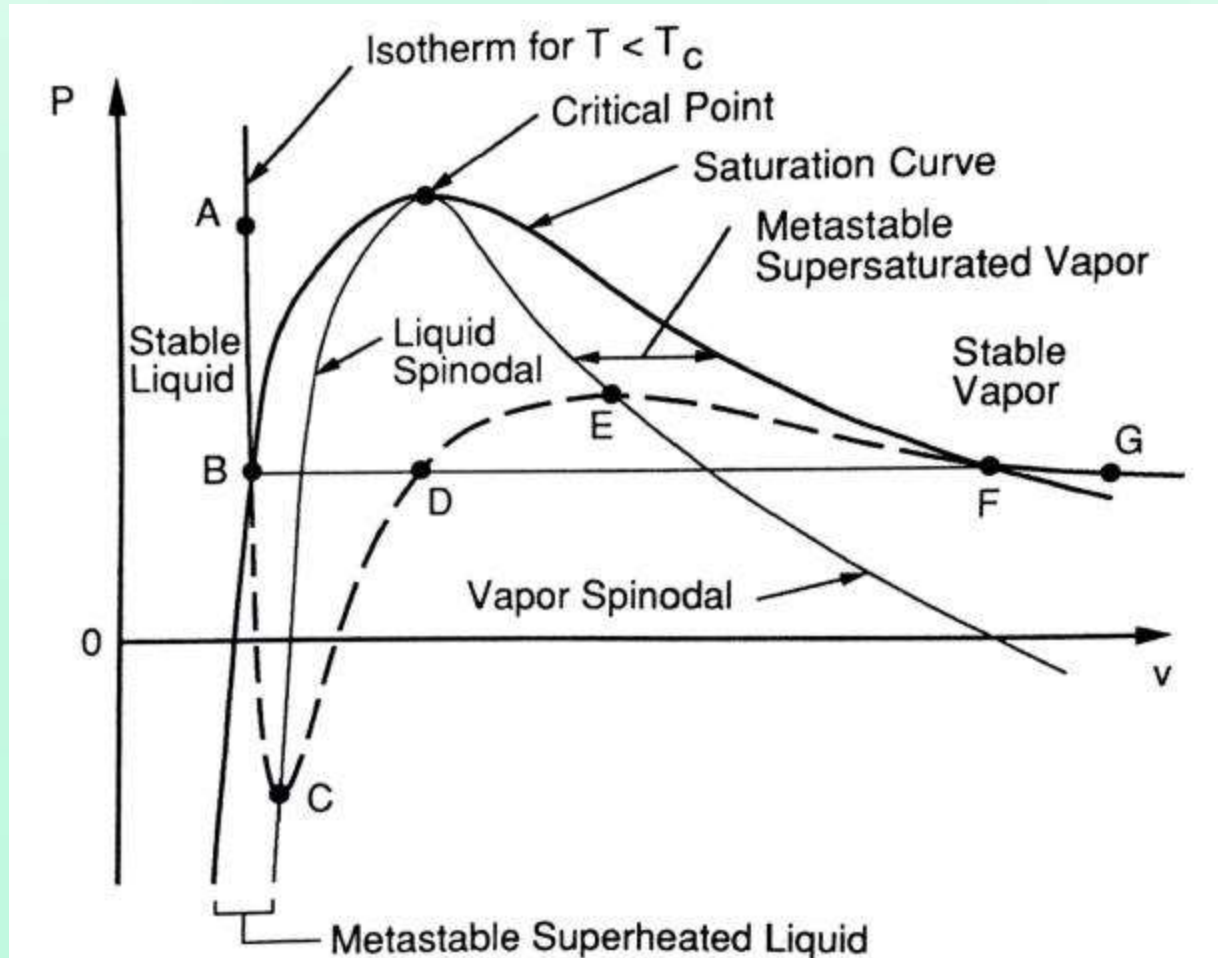
# Metastable State



# Compressing water (I) isothermally



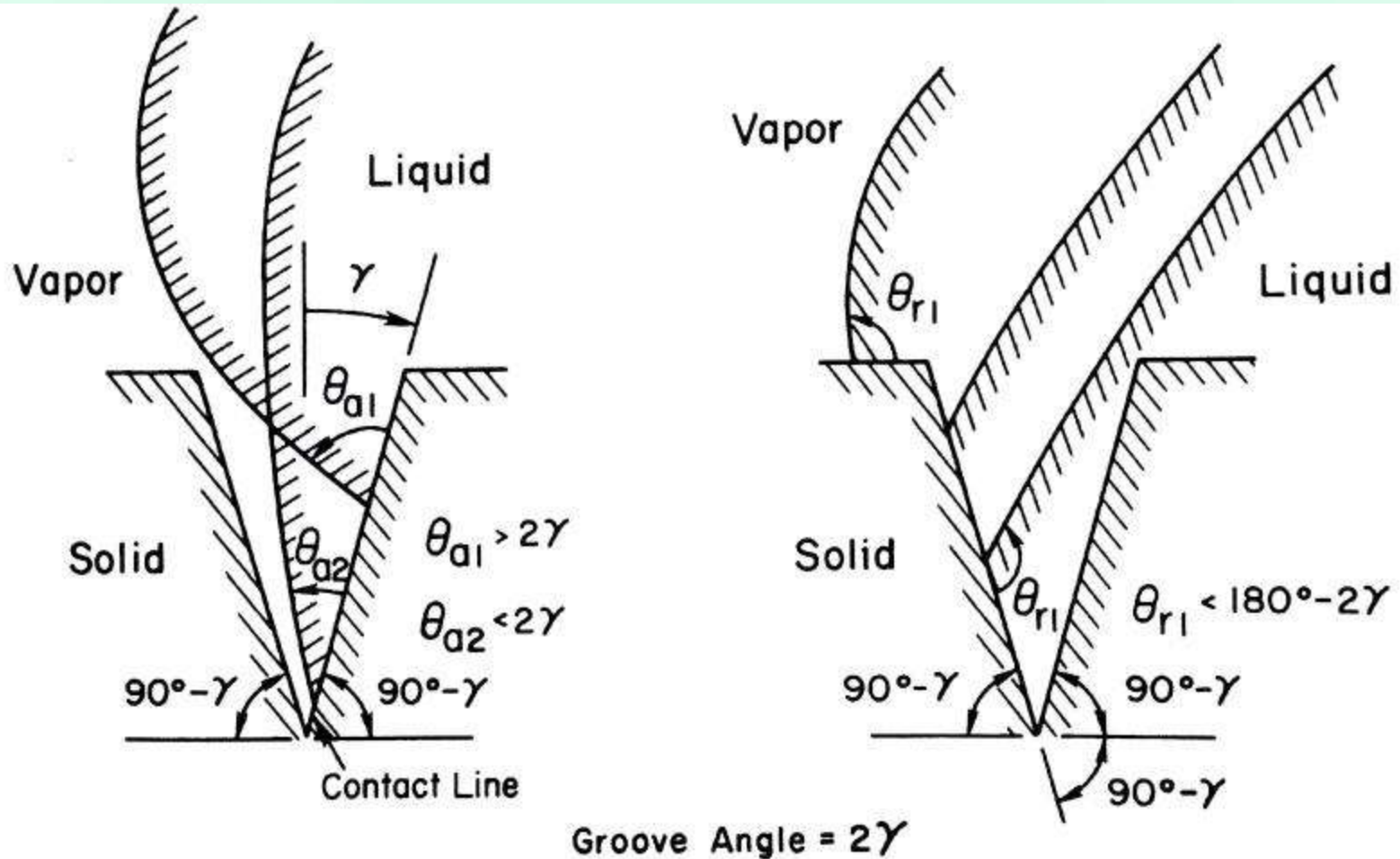
# Metastable Regions – Spinodal Points



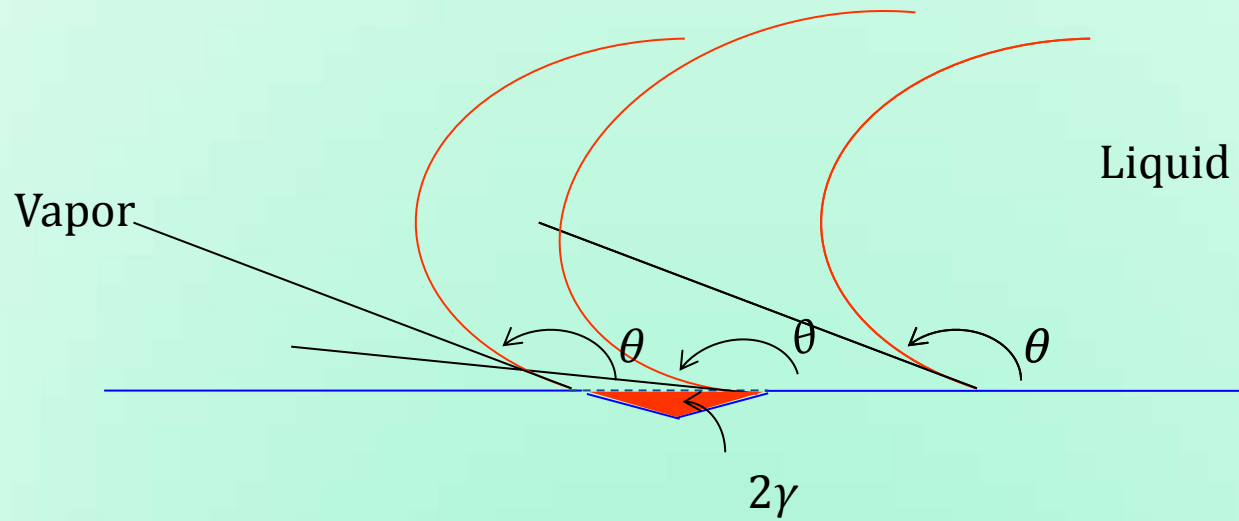
# Homogeneous and Heterogeneous Nucleation

- Bubble nucleation completely within a superheated liquid or subcooled vapor is called homogeneous nucleation.
- Nucleation at the interface between a metastable phase and another phase that it contacts is called heterogeneous nucleation.

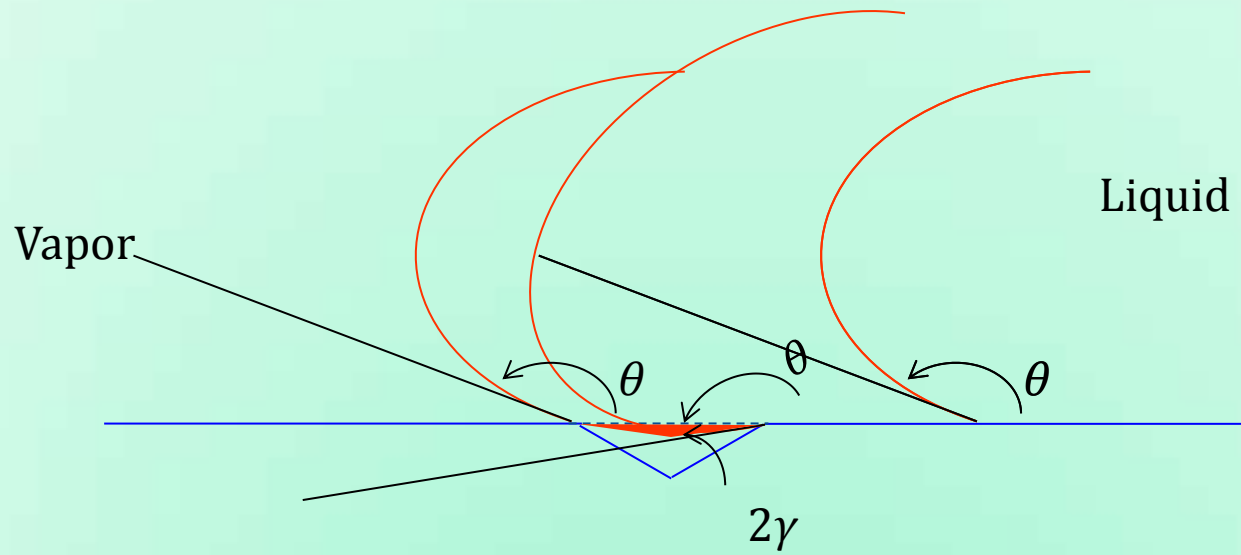
# Trapped Vapor or Liquid Pocket



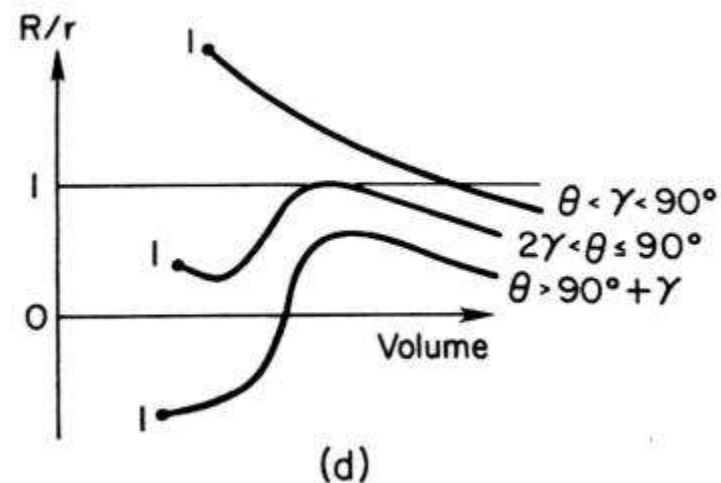
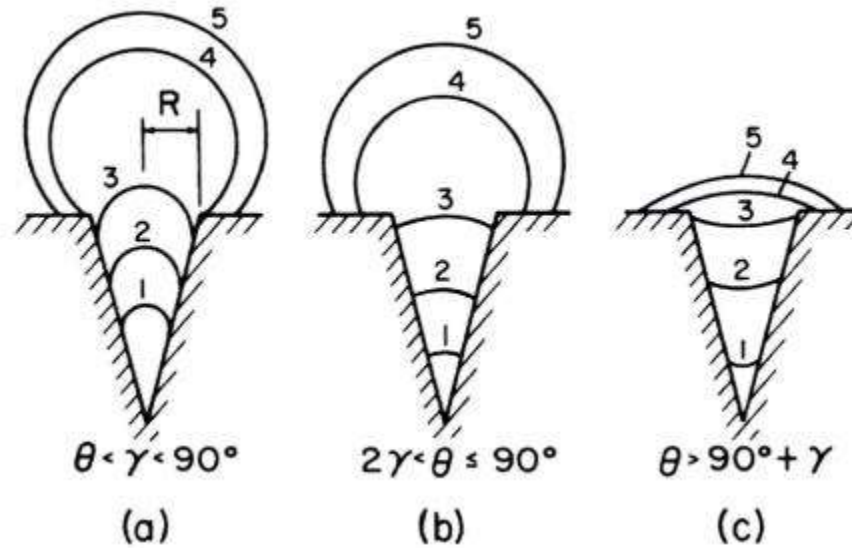
# Trapped Vapor Pocket



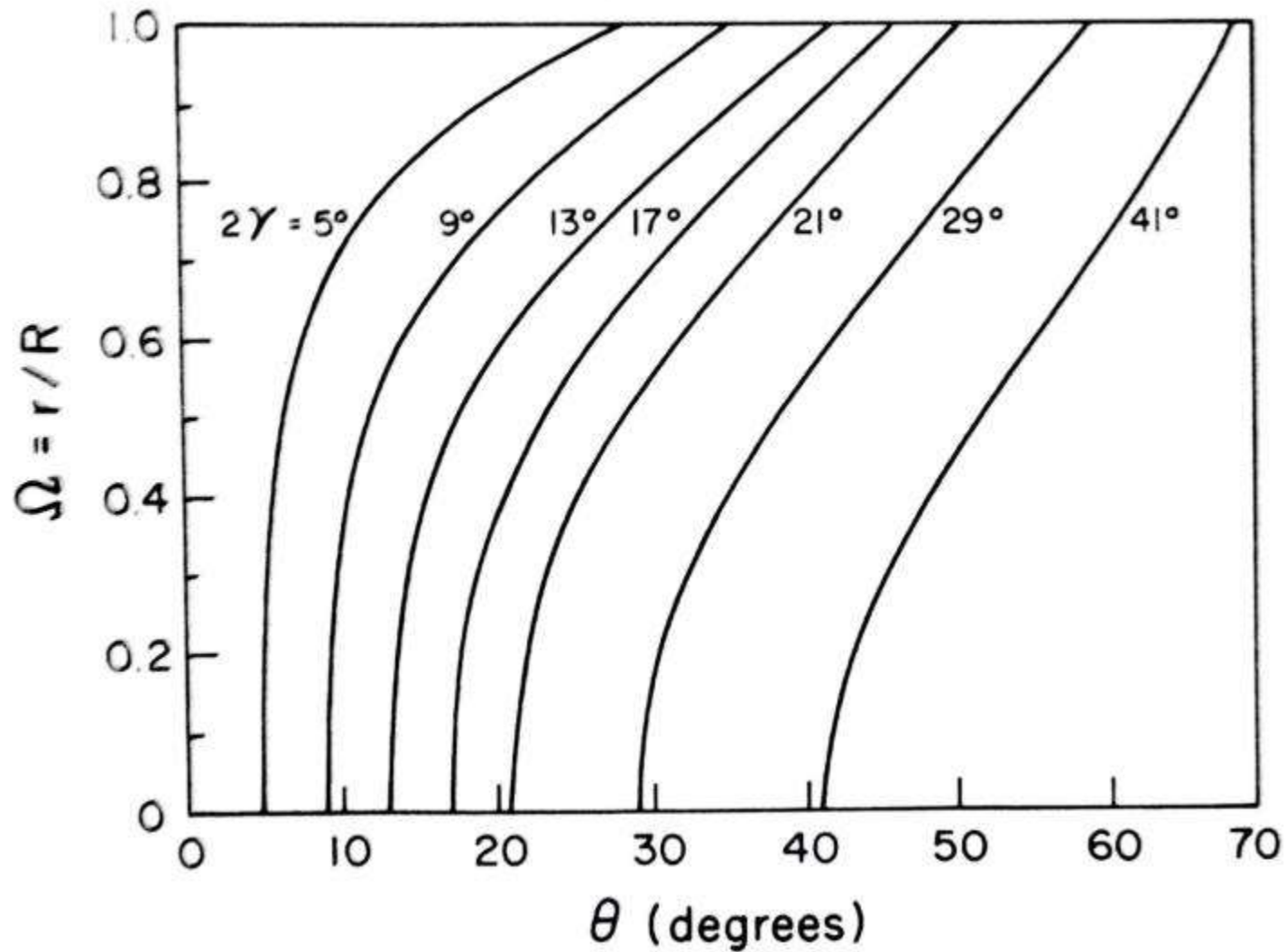
# Trapped Vapor Pocket



# Bubble $r$ as Bubble Grows within and out of Cavity



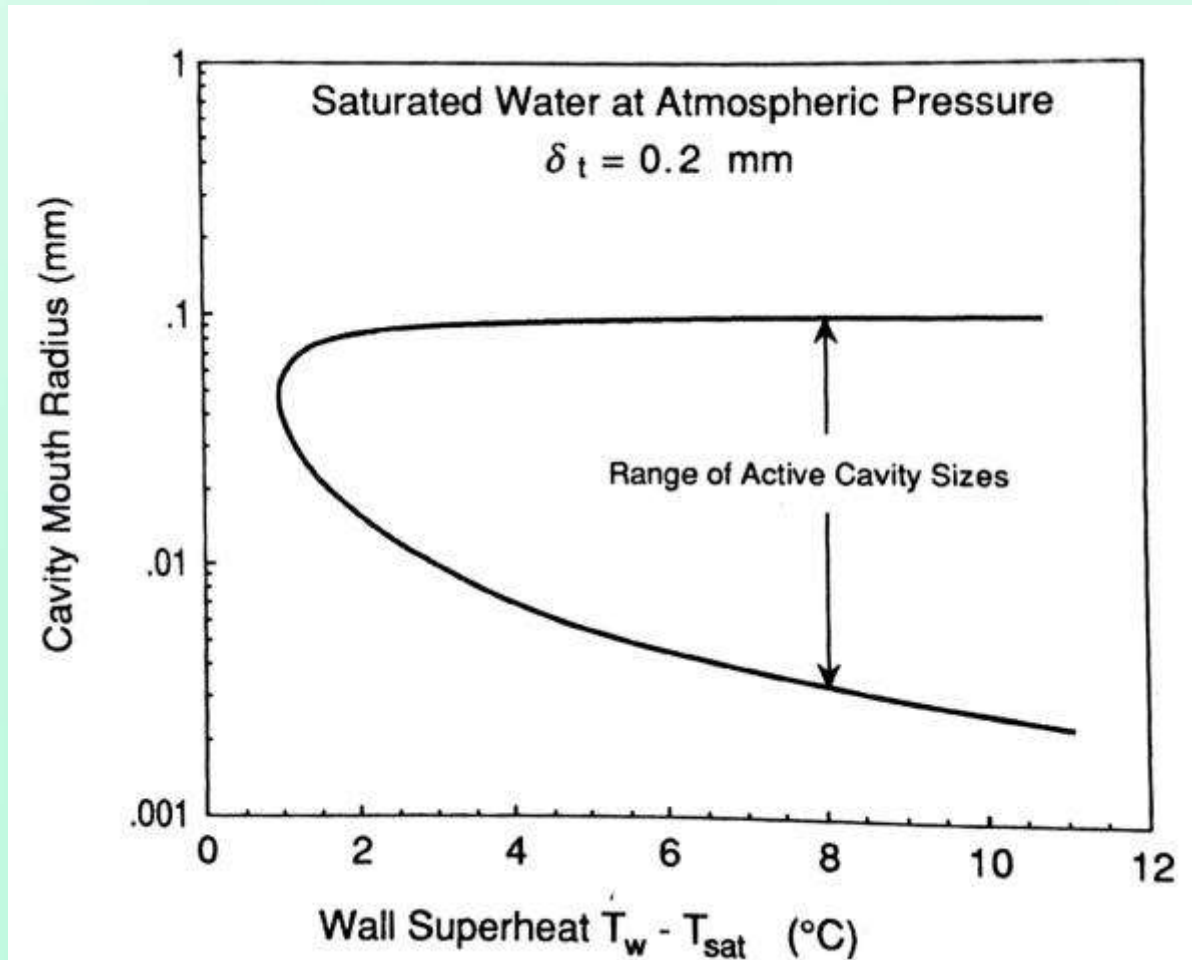
# Initial $r$ of Vapor Embryo with Cavity Cone Angle ( $\gamma$ ), $\theta$



# Heterogeneous Nucleation

1. A certain minimum value of wall superheat must be attained before any cavities on the surface will become active nucleation sites
2. Above the superheat required to initiate nucleation, a finite range of cavities can become active sites. The extent of this range depends on the fluid properties, thermal boundary layer, thickness ( $\delta_t$ ) and the subcooling of the bulk fluid.

# Heterogeneous Nucleation



# Terminology

1. Ebullition cycle: The steady cyclic growth and release of vapor bubbles at any active nucleation site
2. Bubble departure ( $d_d$ ) at departure
3. Waiting period ( $t_w$ ): The portion of the bubble growth and release cycle
4. Frequency at which bubbles are generated and released

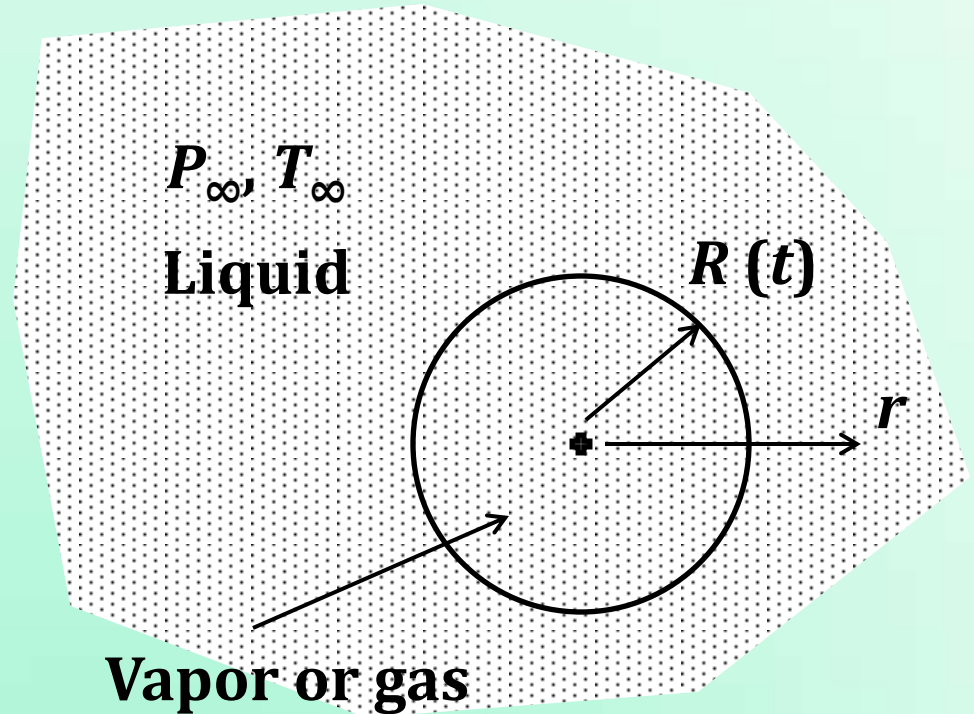
Time period ( $\tau$ ) associated with the growth of each bubble, must equal the sum of the waiting period and the time required for the bubble grow to its departure diameter.

$$\frac{1}{f} = \tau = t_w + t_{2R(t)=d_d}$$

# Bubble Growth in a Superheated Liquid

## Assumption

A single bubble of spherical shape



Growing Bubble in  
the bulk of a Liquid

## Some characteristics of Bubble Growth

- Initially, the interface temperature close to superheated liquid temperature
- Vapor generated at interface at pressure nearly equal to  $P_{sat}(T_\infty)$ .
- As the temperature of the superheated liquid near the interface reduces with time (Thermal energy is consumed to generate vapor) the liquid temperature reduces towards  $T_{sat}(P_\infty)$ .
- Pressure inside the bubble is high initially, drops gradually.
- As the radius increases & capillary pressure difference decreases.

## Some characteristics of Bubble Growth

Temperature & Pressure ranges during growth period are:

$$P_{\infty} \leq P_v \leq P_{sat}(T_{\infty})$$

$$T_{sat}(P_{\infty}) \leq T_v \leq T_{\infty}$$

# Rate of Bubble Growth Dictated by

1. Fluid momentum & Pressure difference
2. Rate of Vaporization which depends on rate of heat transfer
3. Local thermodynamic equilibrium which is assumed to exist at the interface

$$P_v = P_{sat}(T_v)$$

# Two Limiting Cases of Bubble growth

## 1. Inertia Controlled Bubble Growth

- Initial stage of growth
- Pressure has the Maximum value
- $P_{sat}(T_\infty) \& T_v \cong T_\infty$
- Growth rate dictated by momentum transfer, not by rate of heat transfer
- Faster rate of growth

## 2. Heat transfer controlled Bubble growth

- Later stage of Bubble growth
- $T_v$  approaches Minimum value
- $T_v \rightarrow T_{sat}(P_\infty)$  and  $P_v \cong P_\infty$
- Growth is dictated by heat transfer (Energy eq.)
- Slow process

# Bubble Radius in Inertia Controlled Growth

Initial stage (Inertia controlled):

$$R(t) = \left\{ \frac{2 [T_\infty - T_{sat}(P_\infty)] h_{lv} \rho_v}{3 T_{sat}(P_\infty) \rho_l} \right\}^{\frac{1}{2}} t$$

Later stage (heat transfer controlled):

$$R(t) = 2 \sqrt{\frac{3}{\pi} \alpha_l t} \text{ Ja}$$

$$\text{Jakob number, Ja} = \frac{\rho_l c_{pl} (T_\infty - T_{sat})}{\rho_v h_{lv}}$$

# Bubble Departure Diameter Correlations

Departure diameter in terms of Bond number

$$\mathbf{Bo}_d^{0.5} = \mathbf{0.04 Ja}$$

$$\text{Jakob number, Ja} = \frac{\rho_l c_{pl} (T_w - T_{sat}(P_\infty))}{\rho_v h_{lv}}$$

$$\text{Bond number, } \mathbf{Bo}_d = \frac{g(\rho_l - \rho_v) d_d^2}{\sigma}$$

Zuber suggested

$$f d_d = \mathbf{0.59} \left[ \frac{\sigma g(\rho_l - \rho_v)}{\rho_l^2} \right]^{0.25}$$

# Inertia-Controlled Bubble Growth

Assuming inertia-controlled growth, estimate the interface velocity of a 0.2 mm diameter bubble growing in water at atmospheric pressure and 120°C.

$$T_{sat} = 100^\circ\text{C}, h_{lv} = 2257 \text{ kJ/kg}, \rho_l = 958 \text{ kg/m}^3, \rho_v = 0.598 \text{ kg/m}^3$$

$$R(t) = \left\{ \frac{2 [T_\infty - T_{sat}(P_\infty)] h_{lv} \rho_v}{3 T_{sat}(P_\infty) \rho_l} \right\}^{\frac{1}{2}} t$$

$$V = 7.1 \text{ m/s}$$

# Bubble Departure Frequency

Estimate the bubbling frequency for saturated water at atmospheric pressure for a wall superheat of 20°C.

$$T_{sat} = 100^\circ\text{C}, h_{lv} = 2257 \text{ kJ/kg}, \rho_l = 958 \text{ kg/m}^3, \rho_v = 0.598 \text{ kg/m}^3,$$

$$\sigma = 0.0588 \text{ N/m}$$

$$\mathbf{Bo}_d^{0.5} = \mathbf{0.04 Ja}$$

$$Ja = \frac{\rho_l c_{pl} (T_w - T_{sat}(P_\infty))}{\rho_v h_{lv}}$$

$$Bo_d = \frac{g(\rho_l - \rho_v) d_d^2}{\sigma}$$

$$f d_d = 0.59 \left[ \frac{\sigma g(\rho_l - \rho_v)}{\rho_l^2} \right]^{0.25}$$

$$\begin{aligned} Ja &= 60 \\ d_d &= 6 \text{ mm} \\ f &= 15.4 \text{ Hz} \\ \tau &= 0.065 \text{ s} \end{aligned}$$

Estimate the interface velocity of a 0.2 mm diameter bubble growing in water at atmospheric pressure and 120°C.

Estimate the departing diameter and bubble frequency.

$$T_{sat} = 100^\circ\text{C}, h_{lv} = 2257 \text{ kJ/kg}, \rho_l = 958 \text{ kg/m}^3, \rho_v = 0.598 \text{ kg/m}^3$$

$$R(t) = \left\{ \frac{2 [T_\infty - T_{sat}(P_\infty)] h_{lv} \rho_v}{3 T_{sat}(P_\infty) \rho_l} \right\}^{\frac{1}{2}} t$$

$$\mathbf{Bo}_d^{0.5} = \mathbf{0.04 Ja}$$

$$\mathbf{Bo}_d = \frac{g(\rho_l - \rho_v) d_d^2}{\sigma} \quad \mathbf{Ja} = \frac{\rho_l c_{pl} (T_w - T_{sat}(P_\infty))}{\rho_v h_{lv}}$$

$$f d_d = \mathbf{0.59} \left[ \frac{\sigma g(\rho_l - \rho_v)}{\rho_l^2} \right]^{0.25}$$

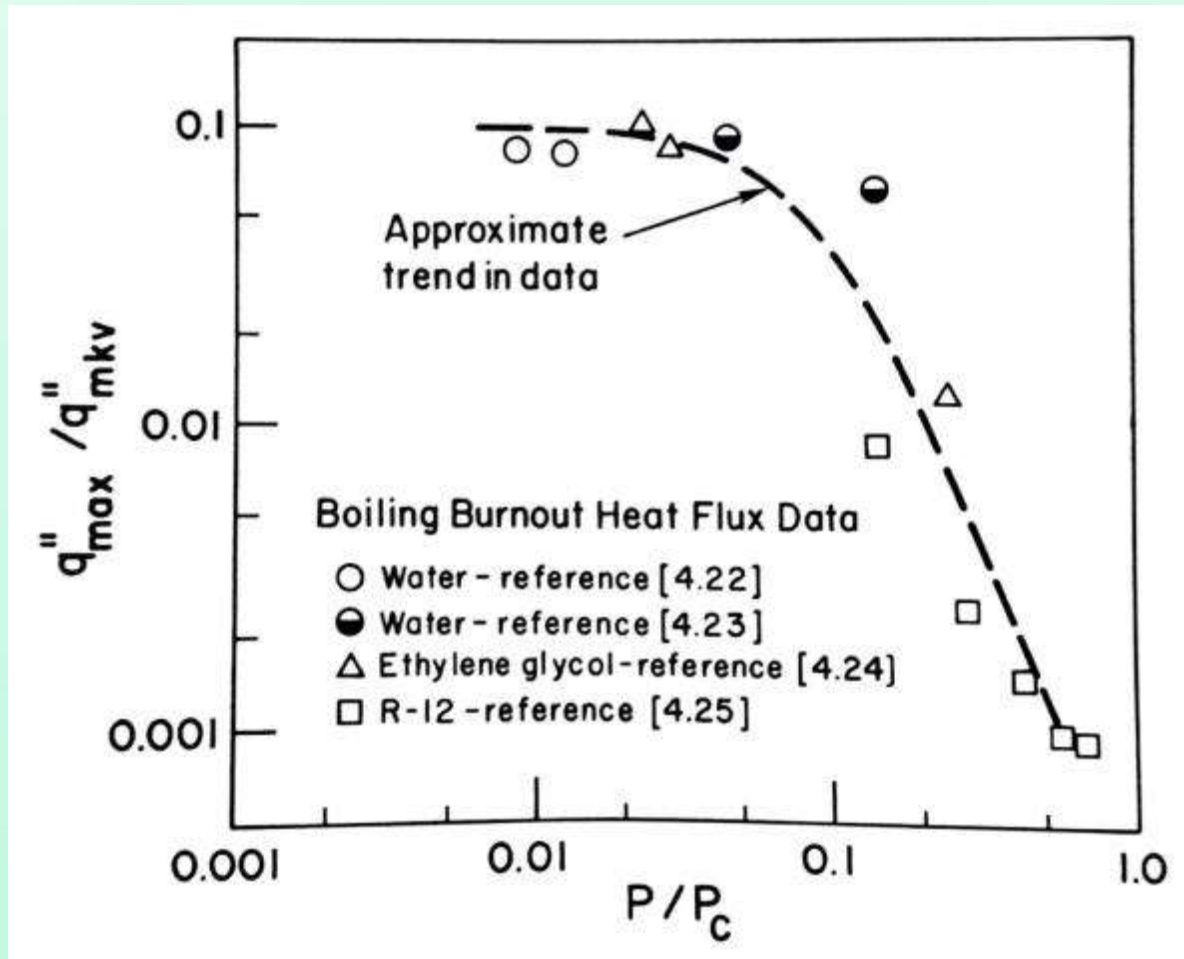
$$V = 7.1 \text{ m/s}$$

# Kinetic Theory - Maximum Flux Limitations

- There is an upper limit to the heat flux that can be attained in a phase-change process
- For a vaporization process, the maximum heat flux conceivable would result if molecules were emitted from the interface and no molecules were allowed to return to the liquid.
- Assuming Maxwellian ideal gas,

$$q''_{mkv} = \rho_v h_{lv} \left( \frac{\overline{RT}_v}{2\pi\overline{M}} \right)^{\frac{1}{2}}$$

# Maximum Flux vs Burnout



Measured burnout data obtained in different boiling experiments normalized with the corresponding maximum heat flux suggested by kinetic theory

# Reasons for Deviation

1. Many molecules leaving the liquid at the interface will return to the liquid by molecular collisions
  2. Wall temperatures required to deliver even 10% of  $q''_{mkv}$  would result in wall temperatures that exceed the melting point of the material
  3. The tendency for more of the heat to find single-phase convection and conduction paths at higher pressures
- Same tendency is applied for condensation but with a different constant

$$q''_{mkv} = 0.741 \rho_v h_{lv} \left( \frac{\bar{R} T_v}{2\pi \bar{M}} \right)^{\frac{1}{2}}$$

# **Multiphase Flow and Heat Transfer**

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# Condensation

- Condensation is a process in which the removal of heat from a system causes a vapor to convert into liquid.
- Important role in nature:
  - Crucial component of the water cycle
  - Industry
- The spectrum of flow processes associated with condensation on a solid surface is almost a mirror image of those involved in boiling.
- Can also occur on a free surface of a liquid or even in a gas
- Condensation processes are numerous, taking place in a multitude of situations.

# Classification of Condensation Process

1. Mode of condensation: homogeneous, dropwise, film, or direct contact
2. Conditions of the vapor: single-component, multicomponent with all components condensable, multicomponent including non-condensable component(s), etc.
3. System geometry: plane surface, external, internal, etc.

There are overlaps among different classification methods.

Classification based on mode of condensation is the most useful.

# Homogeneous Condensation

- Can happen when vapor is sufficiently cooled  $< T_{sat}$  to induce droplet nucleation.
- It may be caused by:
  - Mixing of two vapor streams at different temperatures
  - Radiative cooling of vapor-noncondensable mixtures
    - **Fog formation**
  - Sudden depressurization of a vapor
    - **Cloud formation** – adiabatic expansion of warm, humid air masses that rise and cool
    - Cloud - water or ice?  $-30^{\circ}\text{C}$
- Although homogeneous nucleation in pure vapors is possible, in practice dust, other particles act as droplet nucleation embryos

# Heterogeneous Condensation

- Droplets form and grow on solid surfaces
- Significant sub-cooling of vapor is required for condensation to start when the surface is smooth and dry.
- The rate of generation of embryo droplets in heterogeneous condensation can be modeled by using kinetic theory
- Heterogeneous condensation leads to:
  1. dropwise condensation
  2. film condensation



*Film Condensation*



*Dropwise Condensation*

# Dropwise vs Film

- **Film:** The surface is blanketed by a liquid film of increasing thickness. “Liquid wall” offers resistance.
- **Dropwise:** The droplets slide down when they reach a certain size, clearing the surface and exposing it to vapor.
- No resistance to heat transfer in dropwise. Hence,  $h$  is 10 times higher than in film.

# Dropwise Condensation

- Drop condensation on the underside of a cooled horizontal plate or on a vertical surface is very analogous to nucleate boiling. Ex: misting up of windows or mirrors.
- The condensate forms droplets that stick to the surface.
- The population of droplets becomes large, run together to form films – transition to film condensation.



# Dropwise Condensation

- Poorly wetted surface: on a solid surface cooled below  $T_{sat}$
- At locations of well-wetted: contaminant nuclei exist.
- Poorly-wetted surface condition: contamination or coating with a hydrophobic substance.
- Droplets grow, fall off, new droplets on freshly exposed surface.
- Sweeping and renewal of the droplet growth process is responsible for the high  $h$ .



# Dropwise Condensation

- In practice, this can be achieved from steam condensation by
  1. Injecting a non-wetting chemical into the vapor which subsequently deposits on the surface
  2. Introducing a non-wetting ( $\theta > 90^\circ$ ) substance such as a fatty acid or wax onto the solid surface
  3. By permanently coating the surface with a low-surface-energy polymer or a noble metal.
- The effects of 1 & 2 are temporary.

# Dropwise Condensation

- Providing and maintaining the non-wetting surface characteristics can be difficult.
- The condensate liquid often gradually removes the promoters.
- Furthermore, the accumulation of droplets on a surface can eventually lead to the formation of a liquid film.

# Dropwise Condensation

- It has been postulated that heat transfer occurs at the smaller droplets due to higher thermal resistance in larger drops.
- Rose *et al.*, (1999) recommended an empirical correlation:

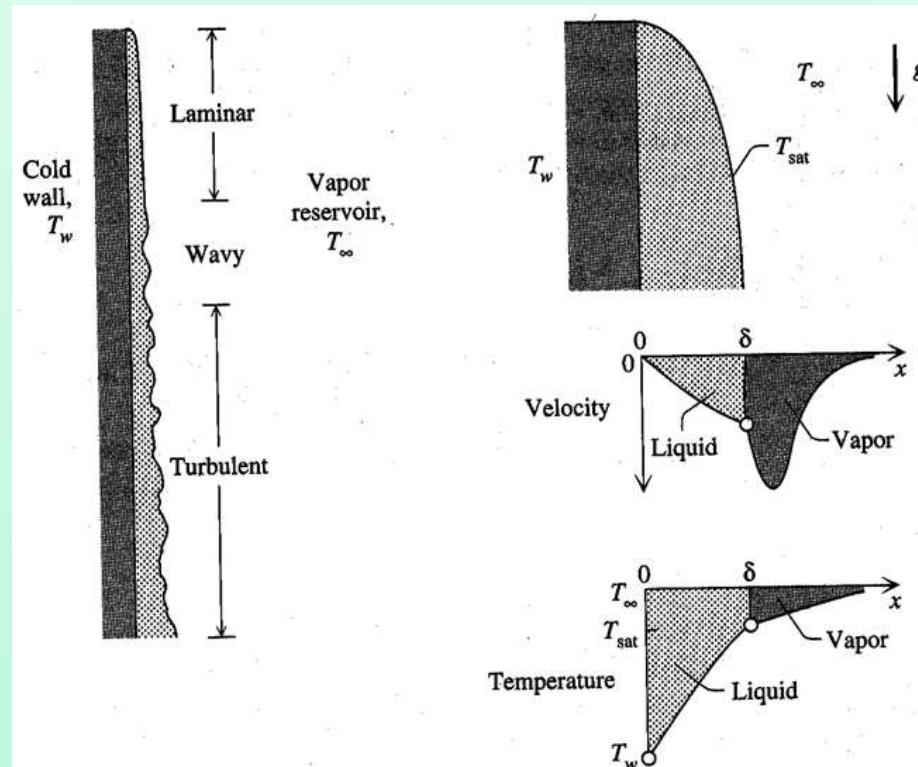
$$h_{dc} = T_v^{0.8} [5 + 0.3(T_{sat} - T_w)]$$

where  $T_v$  is in Celsius, and  $h_{dc}$  is in kW/m<sup>2</sup>K

For a surface subcooling of 8 K, determine the dropwise heat transfer coefficient of saturated steam at atmospheric pressure.

$$h_{dc} = 295 \text{ kW/m}^2\text{K}$$

# Film Condensation on a Flat Vertical Surface

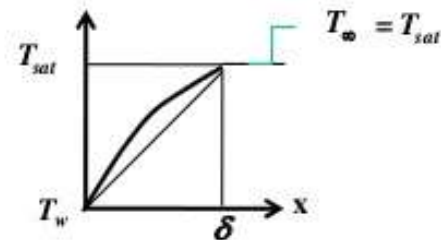
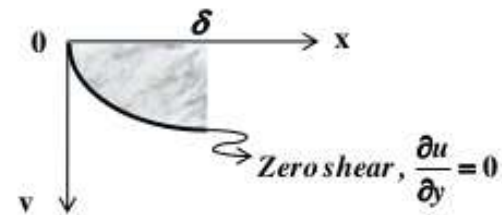
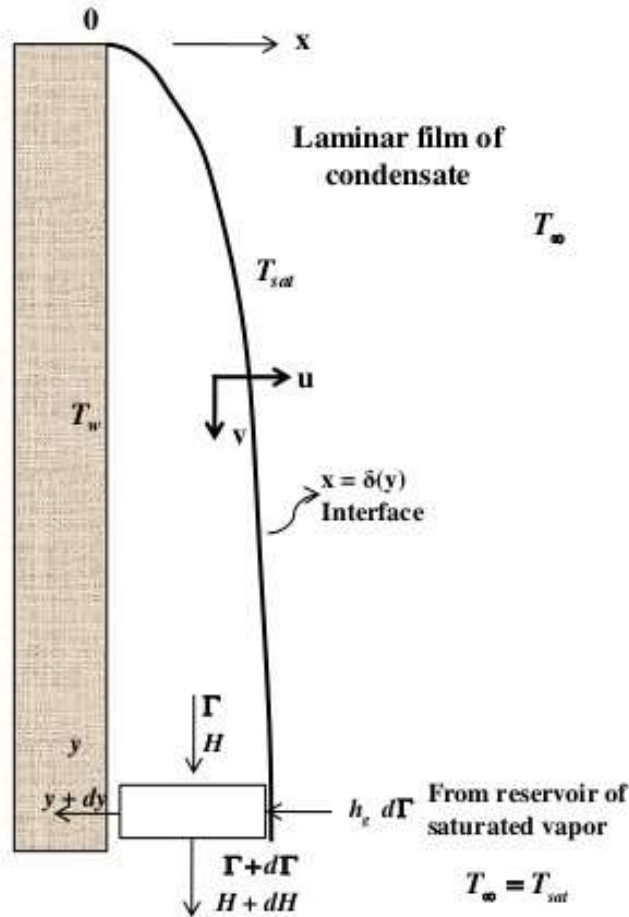


- Temperature of the liquid-vapor interface is the saturation temperature that corresponds to  $T_{sat}$
- Vapor in the descending jet is colder than the vapor reservoir and warmer than the liquid in the film attached to the wall

# Film Condensation on a Flat Vertical Surface

- The wall could be flat or outside surface of a vertical tube
- Consider a vertical wall exposed to a saturated vapor at pressure  $p$  and  $T_{sat} = T_{sat}(p)$
- $T_w < T_{sat}$  vapor will continuously condense on the wall
- If the liquid wets the surface, liquid flows down the wall in a thin film
- Provided the condensation rate is not too large, there will be no discernable waves on the film surface, and the flow in the film will be laminar
  - Fluid dynamics of the flow of a thin liquid film
  - Heat transfer during the flow of a thin liquid film

# Film Condensation on a Flat Vertical Surface



# Nusselt Integral Analysis: Assumptions

- Laminar flow and constant properties
- Inertia effects are negligible in the momentum balance
- Gas is assumed to be pure vapor and at a uniform  $T_{sat}$ 
  - This assumption allows us to focus exclusively on the flow of the liquid film and neglect the movement of the nearest layers of vapor
- Shear stress at the liquid-vapor interface is negligible
- With no temperature gradient in the vapor,
- Heat transfer to the liquid-vapor interface can occur only by condensation at the interface and not by conduction from the vapor

# Steady State 2D Incompressible Flow

Continuity

$$\cancel{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y} = 0 \quad \boxed{\frac{\partial v}{\partial y} = 0}$$

Momentum in x

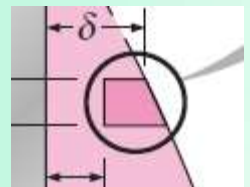
$$\rho_l \left( u \cancel{\frac{\partial u}{\partial x}} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial P}{\partial x} + \mu_l \left[ \frac{\partial^2 u}{\partial x^2} + \cancel{\frac{\partial^2 u}{\partial y^2}} \right]$$

$$\boxed{\frac{\partial P}{\partial x} = 0 \Rightarrow P = P(y)}$$

Momentum in y

$$\rho_l \left( u \cancel{\frac{\partial v}{\partial x}} + v \cancel{\frac{\partial v}{\partial y}} \right) = - \frac{dP}{dy} + \mu_l \left[ \frac{\partial^2 v}{\partial x^2} + \cancel{\frac{\partial^2 v}{\partial y^2}} \right] + \rho_l g$$

$dP/dy$  = Pressure imposed from the inviscid portion  
 =  $\rho_v g$  = Hydrostatic pressure in vapor



$$\mu_l \frac{\partial^2 v}{\partial x^2} + (\rho_l - \rho_v)g = 0$$

Integrate for  $v$

$$v = \frac{(\rho_l - \rho_v)g}{\mu_l} \delta^2 \left[ \frac{x}{\delta} - \frac{1}{2} \left( \frac{x}{\delta} \right)^2 \right]$$

Boundary Conditions

$$x = 0, v = 0$$

$$x = \delta, \frac{\partial v}{\partial x} = 0$$

However, film thickness is an unknown function  $\delta(y)$

The local mass flow rate of the condensate at a location  $y$ , where the boundary layer thickness is  $\delta$ , is determined from:

$$\dot{m}(y) = \int_{x=0}^{\delta} \rho_l v b dx = \frac{\rho_l (\rho_l - \rho_v) g b \delta^3}{3 \mu_l}$$

The rate of condensation of vapor over a vertical distance  $dy$

$$\frac{d\dot{m}}{dy} = \frac{\rho_l (\rho_l - \rho_v) g b \delta^2}{\mu_l} \frac{d\delta}{dy}$$

Flow rate is proportional to sinking effect, inversely proportional to  $\mu$

# Steady State 2D Heat Transfer

Film velocity is low.

$dT$  in  $y$  is negligible since both wall and film surface are isothermal

$$\frac{\partial^2 T}{\partial x^2} = 0$$

$$T = (T_{sat} - T_w) \frac{x}{\delta} + T_w$$

Boundary Conditions

$$x = 0, T = T_w$$

$$x = \delta, T = T_{sat}$$

This is a linear temperature profile similar to the conduction in a plane wall

Heat flux into the wall ( $k_l$ ) = Heat flux across the film ( $h$ )

$$k_l \left. \frac{dT}{dx} \right|_w = h(T_{sat} - T_w)$$

$$h = \frac{k_l}{\delta}$$

Heat transfer across the film is by conduction alone

# Film Thickness, $\delta$

Rate of heat transfer from vapor to the plate through the liquid film  $dy$   
= Heat require to condense the vapor

$$k_l (b dy) \frac{T_{sat} - T_w}{\delta} = d\dot{m} h_{lv}$$

$$\therefore \frac{d\dot{m}}{dy} = \frac{\rho_l (\rho_l - \rho_v) g b \delta^2}{\mu_l} \frac{d\delta}{dy}$$

Solving for  $\delta$  and integrating  $\delta = (0, \delta)$  with  $\delta = 0$  at  $y = 0$

$$\delta(y) = \left[ \frac{4\mu_L k_l (T_{sat} - T_w)}{\rho_L g (\rho_L - \rho_v) h_{lv}} y \right]^{\frac{1}{4}}$$

$$h = \frac{k_l}{\delta} = \left( \frac{\rho_l g (\rho_l - \rho_v) h_{lv} k_l^4}{4 \mu_l k_l (T_{sat} - T_w) y} \right)^{\frac{1}{4}}$$

$$\bar{h}_L = \frac{1}{L} \int_0^L h dy = \left( \frac{g \rho_l (\rho_L - \rho_v) h_{lv} k_l^3}{4 \mu_l (T_{sat} - T_w)} \right)^{\frac{1}{4}} \frac{1}{L} \int_0^L y^{-\frac{1}{4}} dy$$

$$\bar{h}_L = 0.943 \left( \frac{g \rho_l (\rho_L - \rho_v) h_{lv} k_l^3}{4 \mu_l (T_{sat} - T_w) L} \right)^{\frac{1}{4}}$$

$$\dot{m} = \frac{b \rho_l g (\rho_l - \rho_v)}{3 \mu_l} \left[ \frac{4 \mu_l k_l^4 (T_{sat} - T_w)}{\rho_l g (\rho_l - \rho_v) h_{lv}} y \right]^{\frac{3}{4}}$$

All the properties are evaluated at:  $T_f = \frac{T_{sat} + T_w}{2}$

# Effect of subcooling

Rohsenow refined

- Avoided linear temperature profile
- Integral analysis of temperature distribution across the film
- Temperature profile whose curvature increases with the degree of subcooling  $C_{p,l} (T_{sat} - T_w)$

$$h'_{lv} = h_{lv} + 0.68 C_{p,l} (T_{sat} - T_w)$$

All liquid properties evaluated at  $T_f$

$h_{lv}$  and  $\rho_v$  are evaluated at  $T_{sat}$

$$h'_{lv} = h_{lv} (1 + 0.68 \text{Ja})$$

$$\text{Ja} = \frac{C_{p,l} (T_{sat} - T_w)}{h_{lv}}$$

# Reynolds Number

$$\text{Re} = \frac{\rho_l u_m D_h}{\mu_l}; \quad u_m = \frac{\dot{m}}{\rho_L \delta}; \quad D_h = \frac{4A_c}{P} = \frac{4\delta b}{b} = 4\delta$$

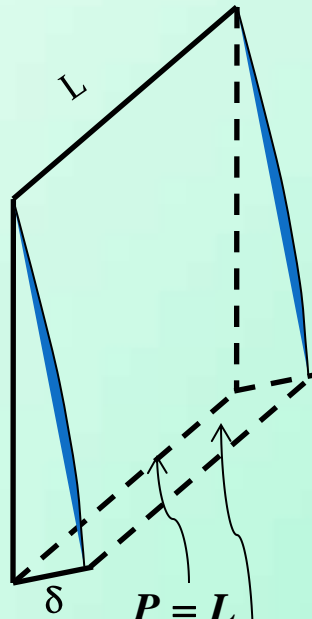
$$\text{Re} = \frac{4\dot{m}}{\mu_l}$$

$$\because \rho_L \gg \rho_v$$

$$\text{Re} = \frac{4\rho_l^2 g \delta^3}{3\mu_l^2} = \frac{4g\delta^3}{3\nu_l^2}$$

$$h_{\text{avg}} = 1.47k_l \text{Re}^{-\frac{1}{3}} \left( \frac{g}{\nu_l^2} \right)^{\frac{1}{3}}$$

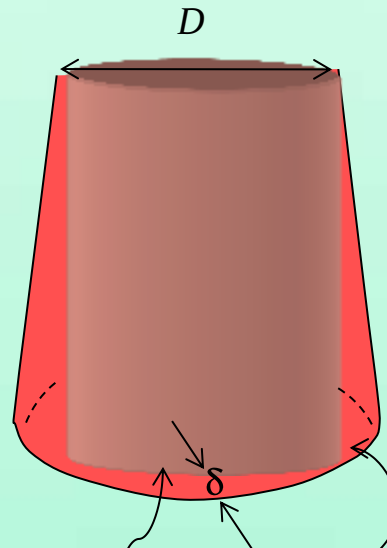
# Hydraulic Diameter



$$P = L$$

$$A_c = L\delta$$

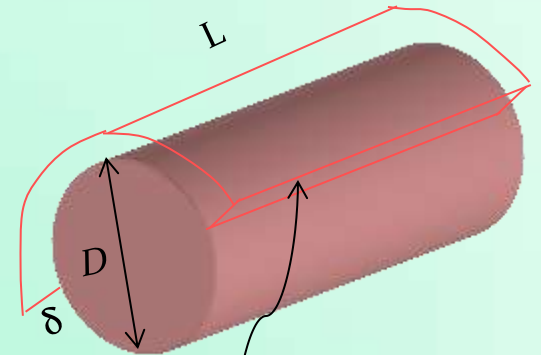
$$Dh = \frac{4A_c}{P} = 4\delta$$



$$P = \pi D$$

$$A_c = \pi D\delta$$

$$Dh = \frac{4A_c}{P} = 4\delta$$



$$P = 2L$$

$$A_c = 2L\delta$$

$$Dh = \frac{4A_c}{P} = 4\delta$$

## Wavy Laminar flow over vertical plates

At Reynolds number greater than about 30, it is observed that waves form at the liquid vapour interface although the flow in liquid film remains laminar. The flow in this case is **Wavy Laminar**

Kutateladze (1963) recommended the following relation for wavy laminar condensation over vertical plates

$$h_{vert,wavy} = \frac{Re k_l}{1.08 Re^{1.22} - 5.2} \left( \frac{g}{\nu_l^2} \right)^{\frac{1}{3}}$$

$$30 < Re < 1800, \rho_v \ll \rho_l$$

$$Re_{vert,wavy} = \left[ 4.81 + \frac{3.70 L k_l (T_{sat} - T_w)}{\mu_l h'_{fg}} \left( \frac{g}{\nu_l^2} \right)^{\frac{1}{3}} \right]^{0.82}$$

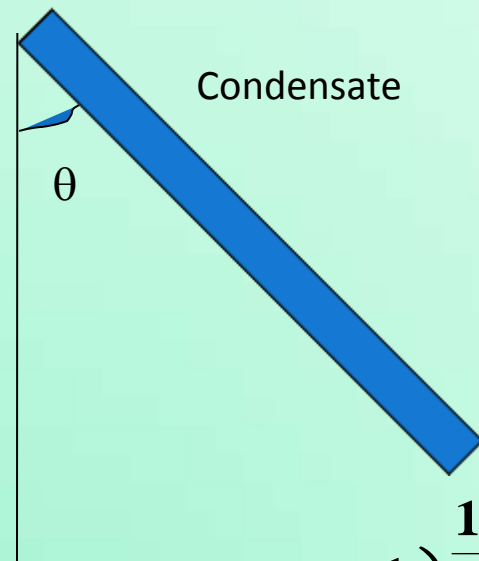
## Turbulent flow over vertical plates ( $Re > 1800$ )

Labuntsov proposed the following relation

$$h_{vert,turbulent} = \frac{Re k_l}{8750 + 58 Pr^{-0.5} (Re^{0.75} - 253)} \left( \frac{g}{\nu_l^2} \right)^{\frac{1}{3}}$$

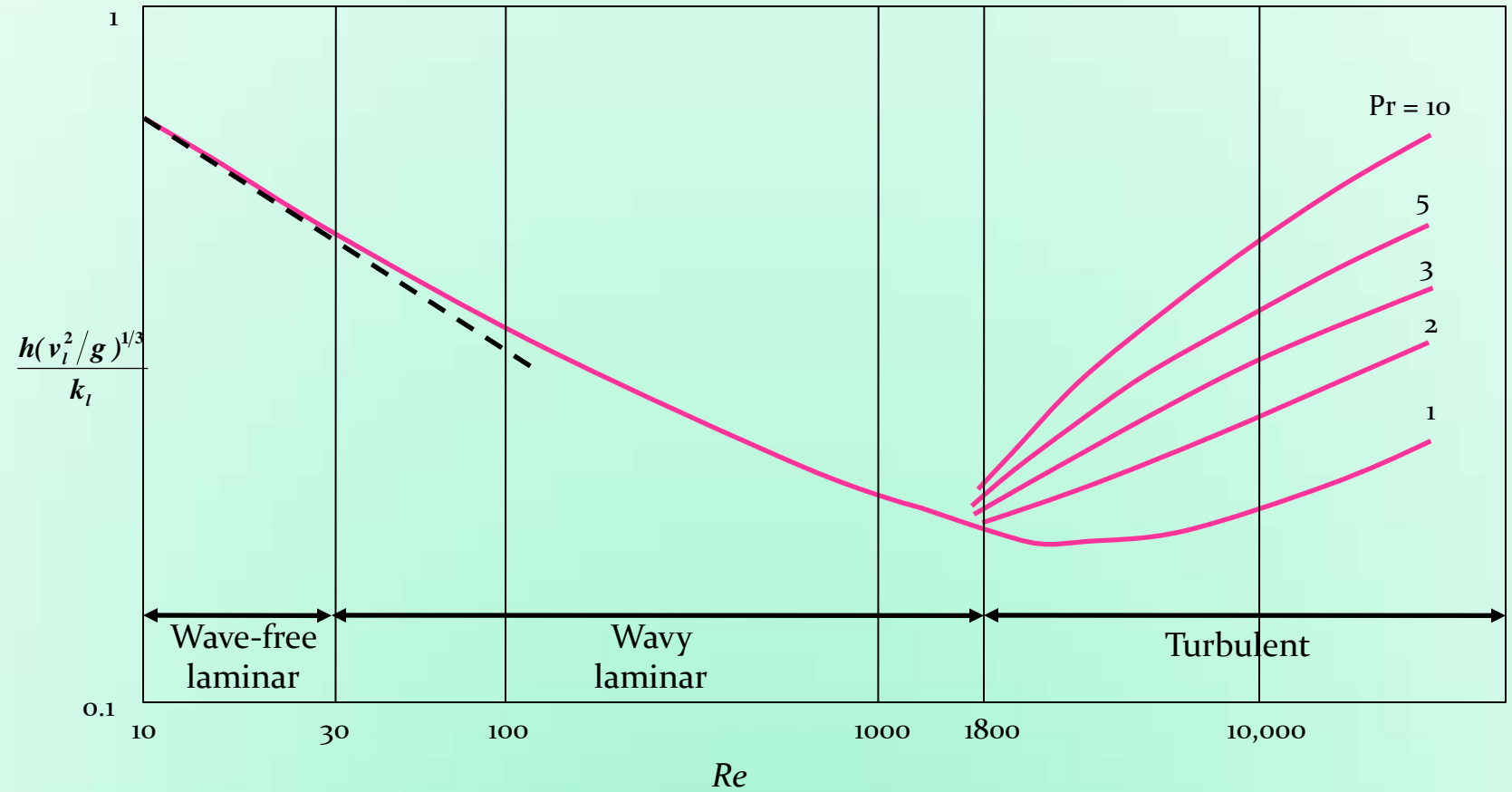
Film condensation on an inclined Plates

$$h_{inclined} = h_{vert} \cos \theta$$



$$\frac{\bar{h}_L}{k_l} \left( \frac{\nu_l^2}{g} \right)^{\frac{1}{3}} = \left( Re_L^{-0.44} + (5.82 \times 10^{-6}) Re_L^{0.88} Pr_L^{\frac{1}{3}} \right)^{\frac{1}{2}}$$

# Non-dimensionalised heat transfer coefficients for the wave-free laminar and turbulent flow of condensate on vertical plates



**Problem:** Saturated steam at atmospheric pressure condenses on a 2 m high and 3 m wide vertical plate that is maintained at 80°C by circulating cooling water through the other side. Determine (a) the rate of heat transfer by condensation to the plate (b) the rate at which the condensate drips off the plate at the bottom

**Solution:** saturated steam at 1 atm condenses on a vertical plate. The rates of heat transfer and condensation are to be determined

**Assumptions:** 1. steady operating conditions exist 2. The plate is isothermal. 3. The condensate flow is wavy laminar over the entire plate (will be verified). 4. The density of vapour is much smaller than the density of the liquid  $\rho_v \ll \rho_l$

**Properties:** The properties of water at the saturation temperature of 100°C are  $h_{fg} = 2257 \times 10^3 \text{ J/g}$  and  $\rho_v = 0.6 \text{ kg/m}^3$ . The properties of liquid water at the film temperature 90°C are

$$T_f = \frac{T_{sat} + T_w}{2} = \frac{100 + 80}{2} = 90$$

$$\rho_l = 965.3 \text{ kg/m}^3$$

$$\mu_l = 0.315 \times 10^{-3} \text{ Pa.s}$$

$$\nu_l = \frac{\mu_l}{\rho_l} = 0.326 \times 10^{-6} \text{ m}^2/\text{s}$$

$$C_{pl} = 4206 \text{ J/kg.K}$$

$$k_l = 0.675 \text{ W/m.K}$$

$$Pr = 1.9628$$

$$h'_{fg} = h_{fg} + 0.68 C_{p,L} (T_{sat} - T_w)$$

$$h'_{fg} = 2257 \times 10^3 + 0.68 \times 4206 \times (100 - 80)$$

$$h'_{fg} = 2314 \times 10^3 \text{ J/kg}$$

$$\bar{h}_L = 0.943 \left( \frac{g\rho_L(\rho_L - \rho_v)h_{fg}k_l^3}{4\mu_L(T_{sat} - T_w)L} \right)^{\frac{1}{4}} = 0.943 \left( \frac{9.81 \times 965.3 \times (965.3)(2314 \times 1000)0.675^3}{4 \times 0.315 \times 10^{-3}(100 - 80)4} \right)^{\frac{1}{4}}$$

$$\bar{h}_L = 26562 \frac{W}{m^2 K}$$

$$\dot{Q} = \bar{h}_L A_s (T_{sat} - T_w) = 25622 \times 2 \times 3 \times (100 - 80) = 307464 W$$

$$\dot{Q} = \dot{m} h_{sf} \Rightarrow 307464 = \dot{m} \times 2314 \times 10^3 \Rightarrow \dot{m} = 0.1329 \text{ kg / s}$$

$$Re = \frac{4\Gamma}{\mu_L} = \frac{4}{\mu_L} \left( \frac{\dot{m}}{b} \right) = \frac{4}{0.315 \times 10^{-3}} \left( \frac{0.1329}{3} \right) = 5625$$

$$\frac{\bar{h}_L}{k_l} \left( \frac{v_l^2}{g} \right)^{\frac{1}{3}} = \left( Re_L^{-0.44} + \left( 5.82 \times 10^{-6} \right) Re_L^{0.88} Pr_L^{\frac{1}{3}} \right)^{\frac{1}{2}}$$

$$\frac{\bar{h}_L}{0.675} \left( \frac{\left( 0.326 \times 10^{-6} \right)^2}{9.81} \right)^{\frac{1}{3}} = \left( 5625^{-0.44} + \left( 5.82 \times 10^{-6} \right) \times 5625^{0.88} \times 1.9628^{\frac{1}{3}} \right)^{\frac{1}{2}}$$

$$\bar{h}_L = 76914 \frac{W}{m^2 K}$$

$$\dot{Q} = \bar{h}_L A_s (T_{sat} - T_w) = 7691.4 \times 2 \times 3 \times (100 - 80) = 2307420 \text{ W}$$

$$\dot{Q} = \dot{m} h_{sf} \Rightarrow 2307420 = \dot{m} \times 2314 \times 10^3 \Rightarrow \dot{m} = 0.9972 \text{ kg / s}$$

$$Re = \frac{4\Gamma}{\mu_L} = \frac{4}{\mu_L} \left( \frac{\dot{m}}{b} \right) = \frac{4}{0.315 \times 10^{-3}} \left( \frac{0.9972}{3} \right) = 4221$$

This confirms that condensation is in turbulent region

**Comments:** This Reynolds number confirms that condensation is in Wavy laminar domain

# **Multiphase Flow and Heat Transfer**

ME546

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# Two Phase Flow

Reference:

S. Mostafa Ghiaasiaan, *Two-Phase Flow, Boiling and Condensation*,  
Cambridge University Press.

<http://dx.doi.org/10.1017/CB09780511619410>

# Two Phase Flow - Introduction

- Two phase flows are commonly found in ordinary life and in industrial processes
- Gas-liquid flow also occurs in boiling and condensation operations
- Inside pipelines which carry oil or gas alone, but which actually carry a mixture of oil and gas.

# Two Phase Flow – How They Differs

## **Single phase flow**

Laminar, transition, and turbulent

When the flow regime changes from laminar to turbulent

the personality of the fluid completely changes

the phenomena governing the transport processes change

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Similar situation

However, there is a multitude of flow regimes

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The behavior of a gas–liquid mixture depends strongly on the flow regimes.

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## **Two phase flow**

Similar situation

However, there is a multitude of flow regimes

The behavior of a gas–liquid mixture depends strongly on the flow regimes.

Methods for predicting the major flow regimes are required, for the modeling and analysis of two-phase flow systems

# Two Phase Flow Patterns

## **Morphological variations**

1.  $\Delta\rho$  between phases. Respond differently to gravity and centrifugal forces

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Flow regimes and their ranges of occurrence are **sensitive** to

- fluid properties, system configuration/and orientation, size scale of the system, occurrence of phase change, etc.

# Two Phase Flow Patterns

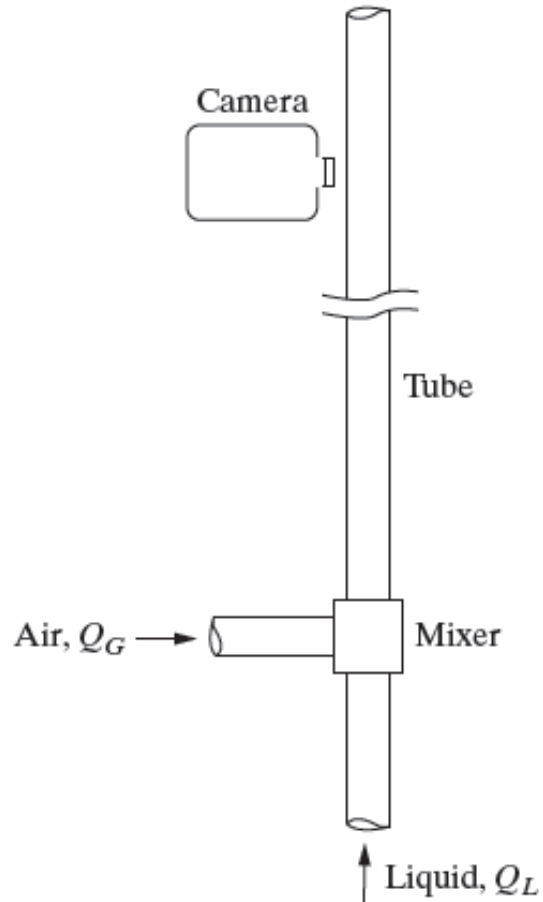
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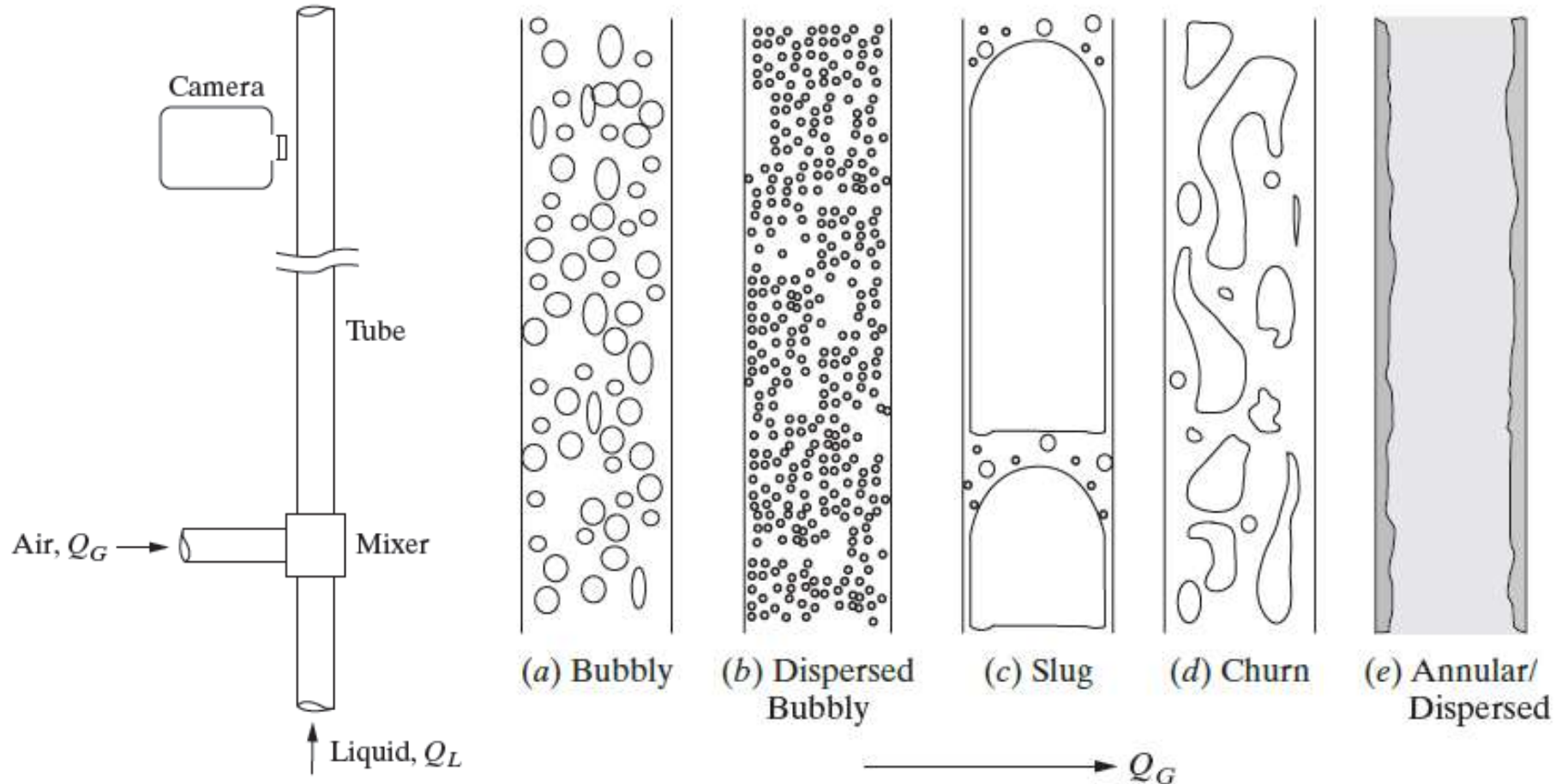
Flow regimes and their ranges of occurrence are **sensitive** to

- fluid properties, system configuration/and orientation, size scale of the system, occurrence of phase change, etc.
- Most widely used: steady-state and adiabatic air-water and steam-water in uniform-cross-section long vertical pipes, or large vertical rod bundles with uniform inlet conditions

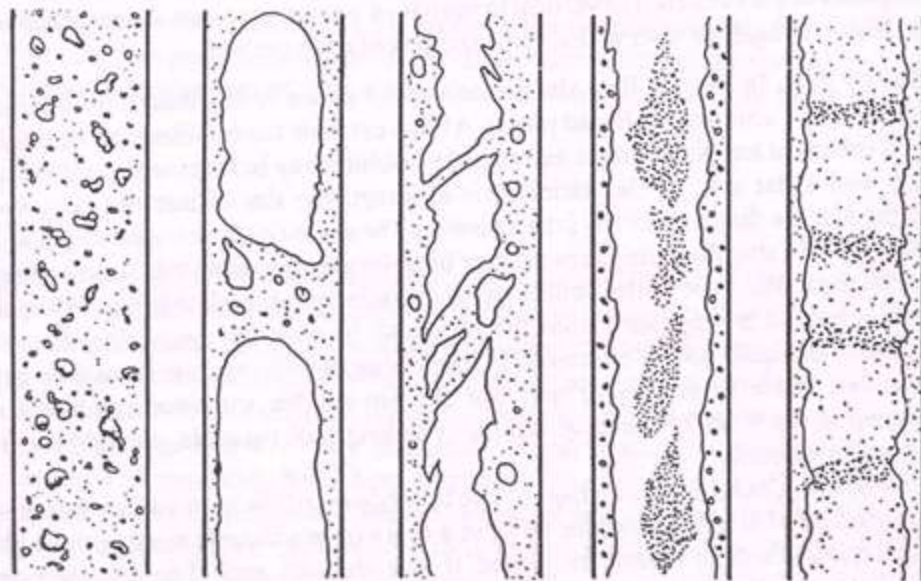
# Vertical, Co-current, Upward Flow



# Vertical, Co-current, Upward Flow



# Vertical Co-current Flow (Adiabatic)



Bubbly

Slug

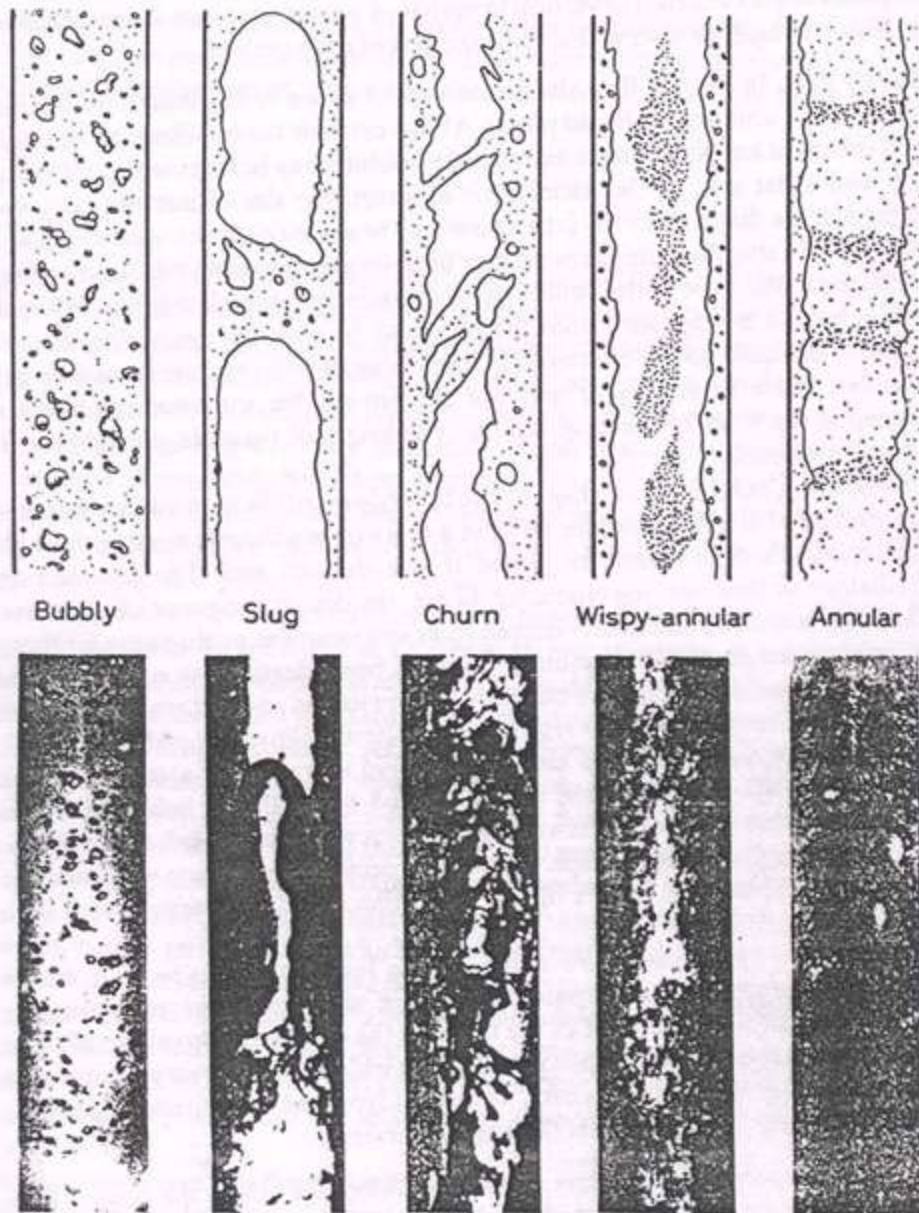
Churn

Wispy-annular

Annular



# Vertical Co-current Flow (Adiabatic)

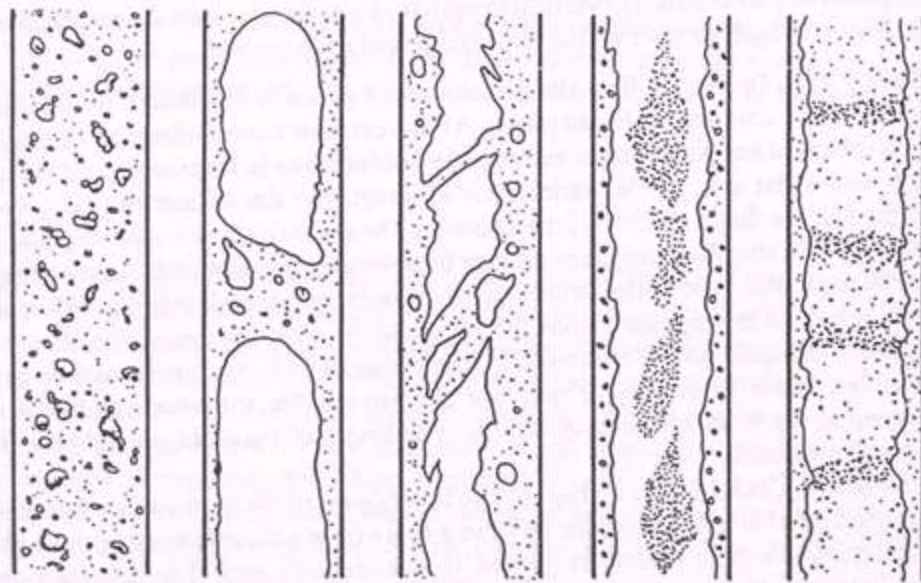


**Bubbly** – bubbles are of uniform size. Least interaction at very low  $Q_G$ , but increase in number density with  $Q_G$ . At higher  $Q_G$ , bubbles interact, leading to their coalescence and breakup.

**Plug/Slug** – Forms very large bubbles. Bullet-shaped (Taylor bubbles) with hemispherical caps and are separated by *liquid slugs* (contains small bubbles). The maximum  $L_s/D \sim 16$ ,

**Churn** – highly unstable/chaotic motion flow of an oscillatory nature, for example the liquid near the tube wall continually pulses up and down.

# Vertical Co-current Flow (Adiabatic)



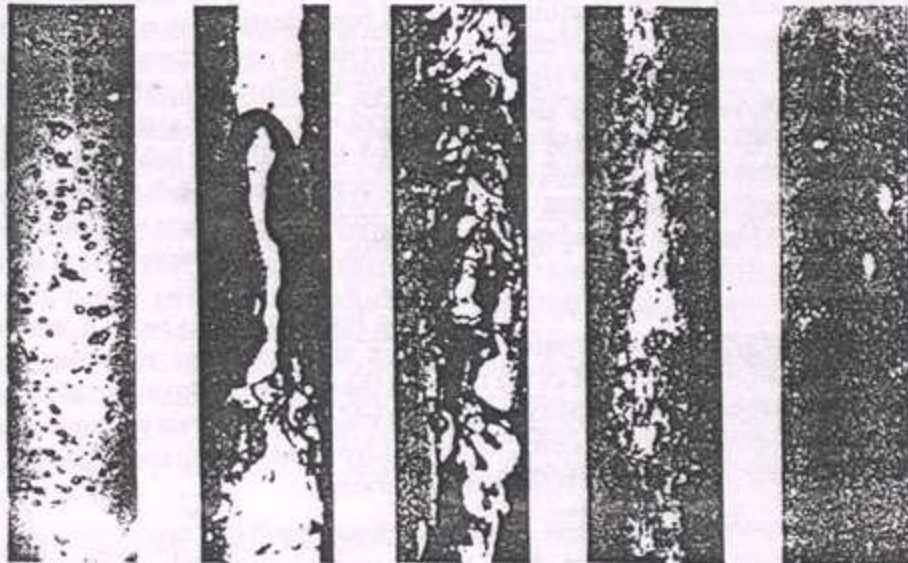
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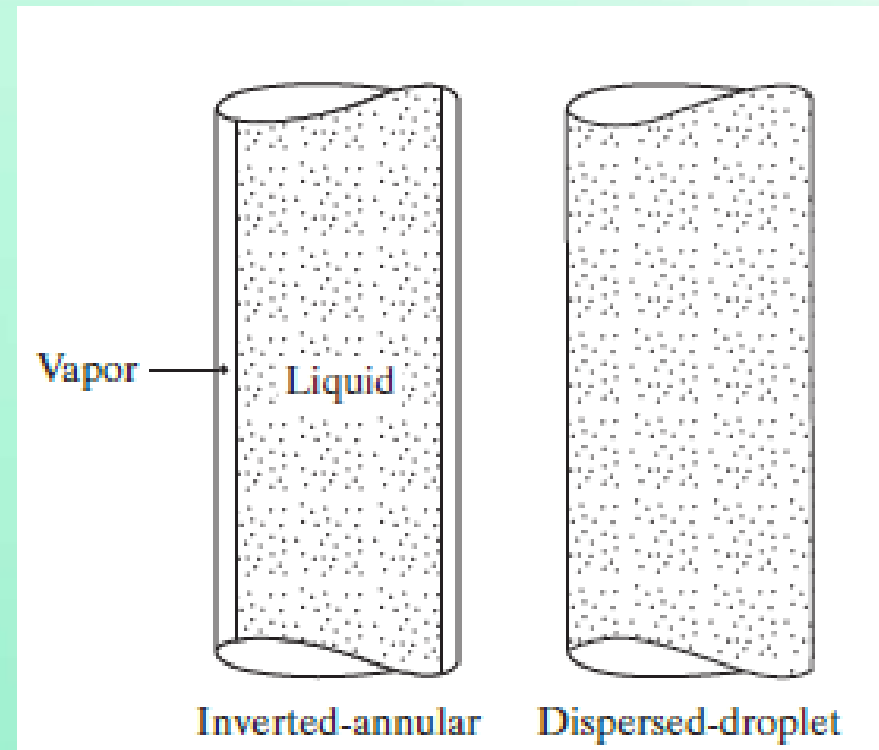
**Wispy annular** - The liquid in the film is aerated by small gas bubbles and the entrained liquid phase appears as large droplets which have agglomerated into long irregular filaments or wisps.

**Annular** - liquid travels partly as an annular film on the walls of the tube and partly as small drops distributed in the gas which flows in the center of the tube

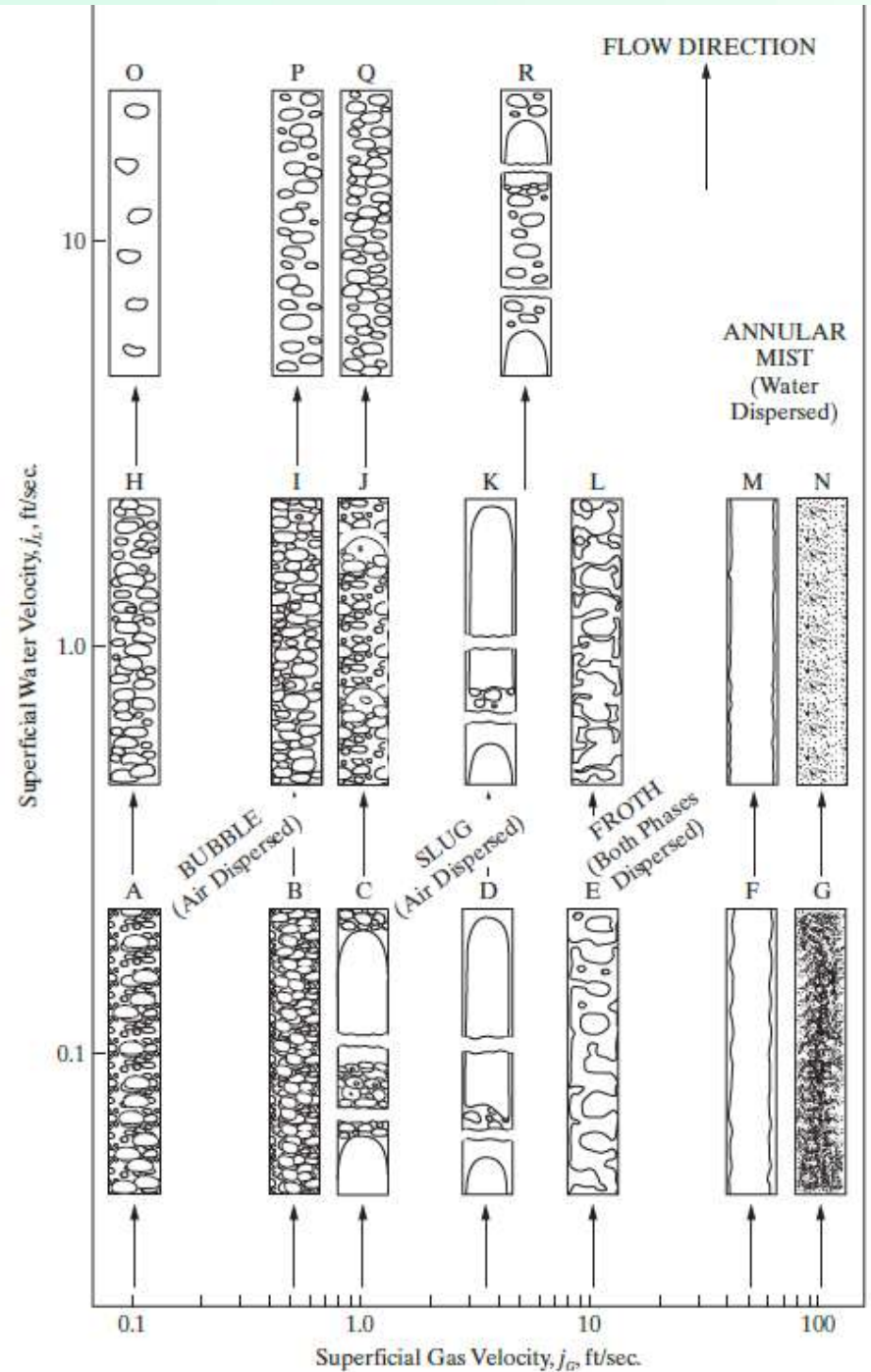
# Vertical Co-current Flow (Boiling Channels)

**Inverted-annular** – This flow regime takes place in channels subject to high wall heat fluxes and leads to an undesirable departure from nucleate boiling.

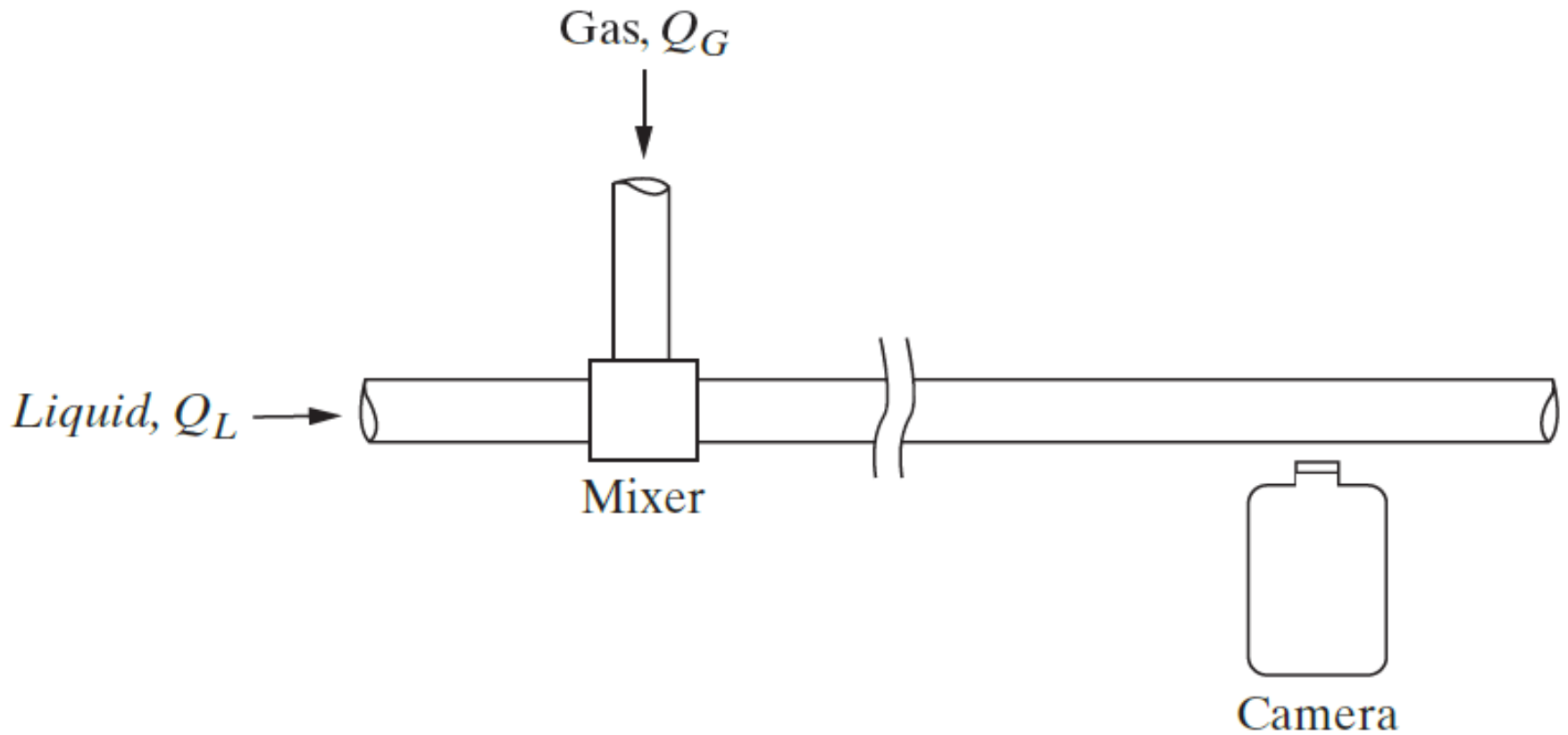
**Dispersed-droplet** – superheated vapor containing entrained droplets flows in a dry channel. Occurs when massive evaporation has already caused the depletion of most of the liquid.



# Flow regimes of air-water flow in a 2.6 cm diameter vertical tube



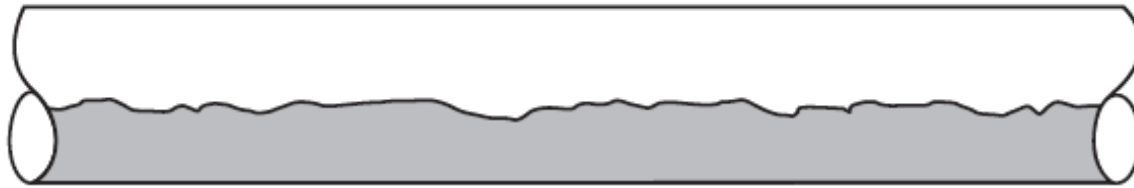
# Horizontal, Co-current



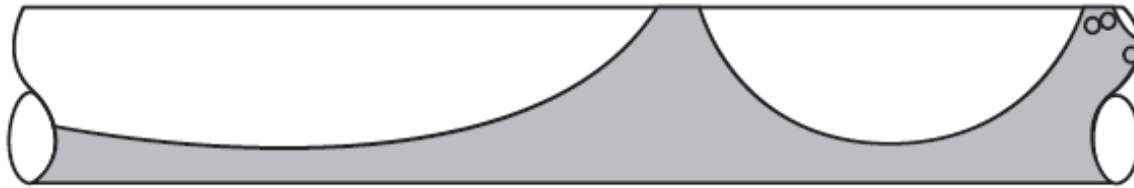
# Horizontal, Co-current – Low Liquid Flow Rate



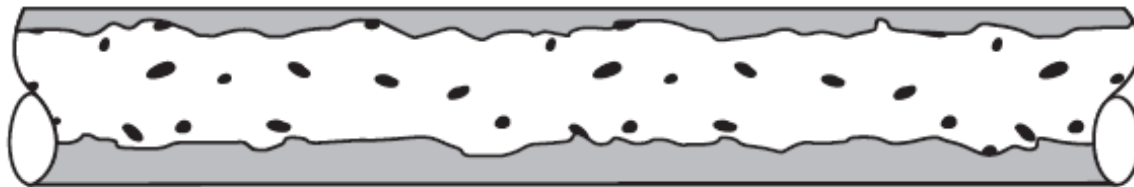
(a) Stratified Smooth



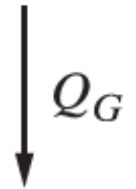
(b) Stratified Wavy



(c) Slug



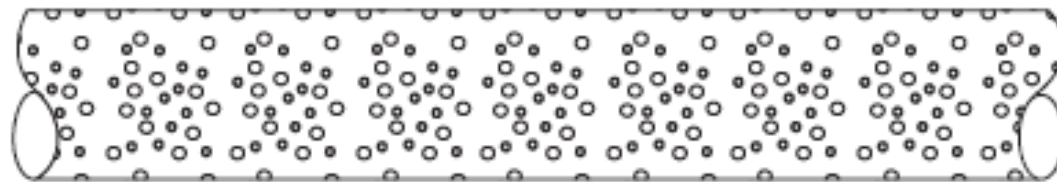
(d) Annular/Dispersed



# Horizontal, Co-current – High Liquid Flow Rate



(a) Bubbly



(b) Dispersed Bubbly



(c) Plug/Elongated Bubble



(d) Annular/Dispersed

# Horizontal Co-current Flow (Adiabatic)

**Bubbly** – bubbles flows on top

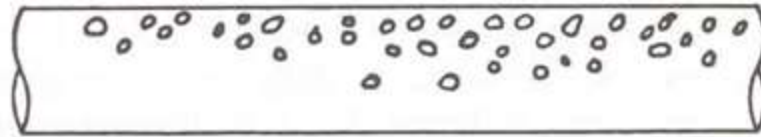
**Plug** – Small bubbles have coalesced to produce long plugs

**Stratified** – interface is smooth.  
This doesn't occur usually

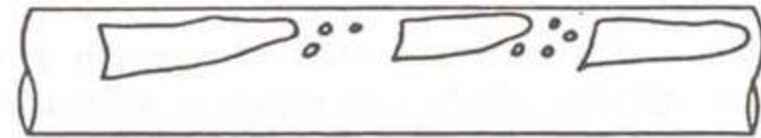
**Wavy** – wave amplitude increases  
as the gas velocity increases

**Slug** – wave amplitude is so large  
that the wave touches top of tube

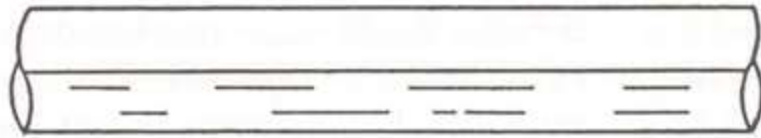
**Annular** – similar to vertical  
annular flow except that the liquid  
film is much thicker at the bottom  
of the tube than at the top.



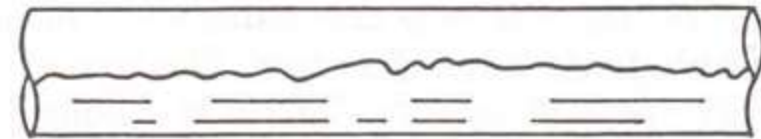
Bubbly flow



Plug flow



Stratified flow



Wavy flow

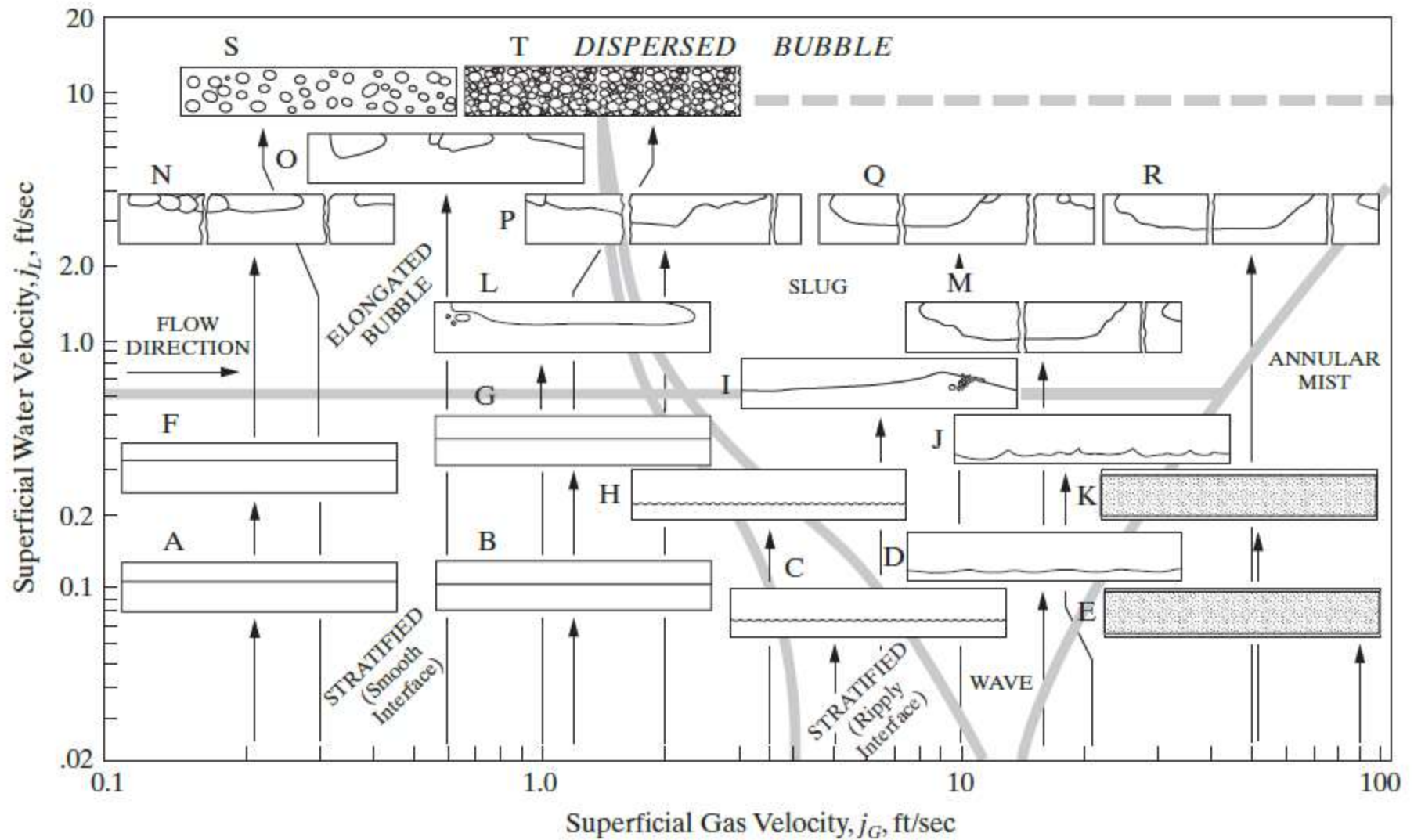


Slug flow

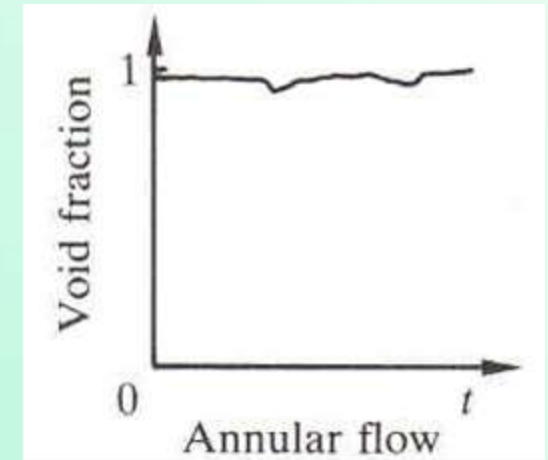
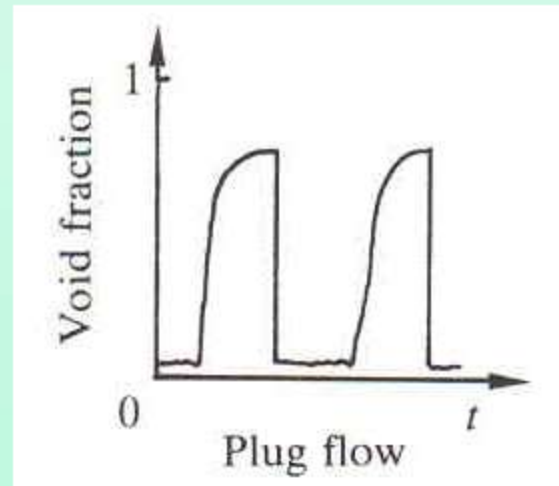
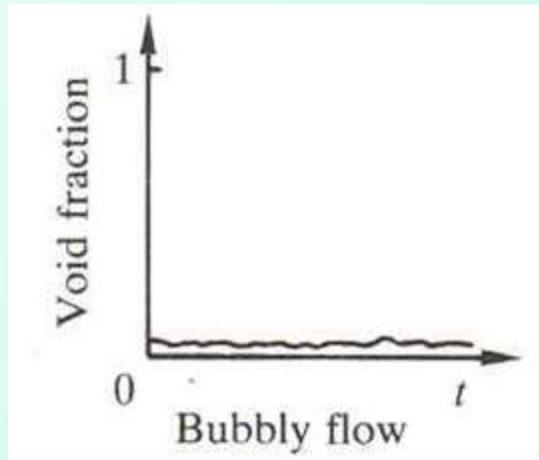


Annular flow

# Horizontal, Co-current



# Idealized Response of a Void Fraction Probe



Various instruments like gamma ray densitometry, capacitance probe and resistance probes give the distribution of void fraction

Results are rarely so conclusive

# Summary

1. Flow regimes and conditions depends on
  - geometry: size, shape, aspect ratio of channel, flow disturbances
  - liquid properties:  $\sigma, \mu, \rho_l/\rho_g$

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5. Bubbly, plug/slug, churn, annular flows also occur in minichannels ( $100 \mu\text{m} \leq D \leq 1 \text{ mm}$ )
6. Regimes in phase change are significantly different from adiabatic

# Flow Pattern Maps

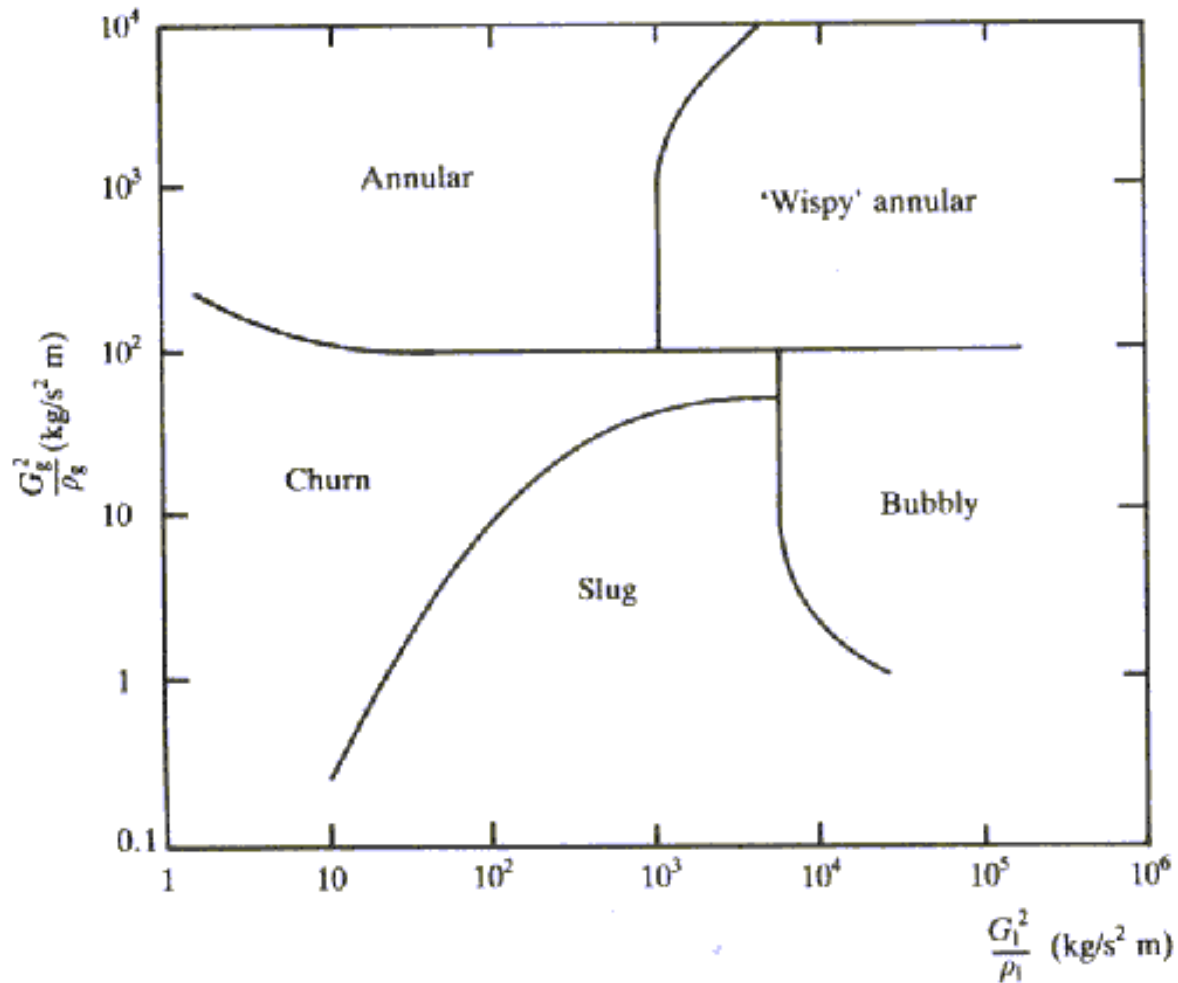
Flow pattern maps are 2D graphs to separate the space into areas corresponding to the various flow patterns

Hewitt and Roberts Map – Vertical upflow in a tube

Baker Map – Horizontal flow

Taitel and Dukler Map – Horizontal flow

# Vertical, Co-current: Hewitt and Roberts



$$G_g = \frac{\text{Gas Mass Flow Rate}}{\text{Tube Crosssectional Area}}$$

$$G_l = \frac{\text{Liquid Mass Flow Rate}}{\text{Tube Crosssectional Area}}$$

This map works reasonably well for water-air and water-steam systems over a range of pressures, again in small diameter tubes

# Baker's Map (1954) - Modified Scott (1963)

- One of the earliest flow pattern maps for horizontal adiabatic flow
- Developed based on air-water data
- Identifies stratified, plug, slug, wavy, annular, bubbly flow patterns

# Procedure to Use Baker's Map (1954)

Determine mass velocities of the liquid ( $G_l$ ) and vapor ( $G_g$ )

Calculate gas-phase parameter  $\lambda$  and liquid-phase parameter  $\psi$

$$\lambda = \left( \frac{\rho_g \rho_l}{\rho_{air} \rho_{water}} \right)^{0.5}$$

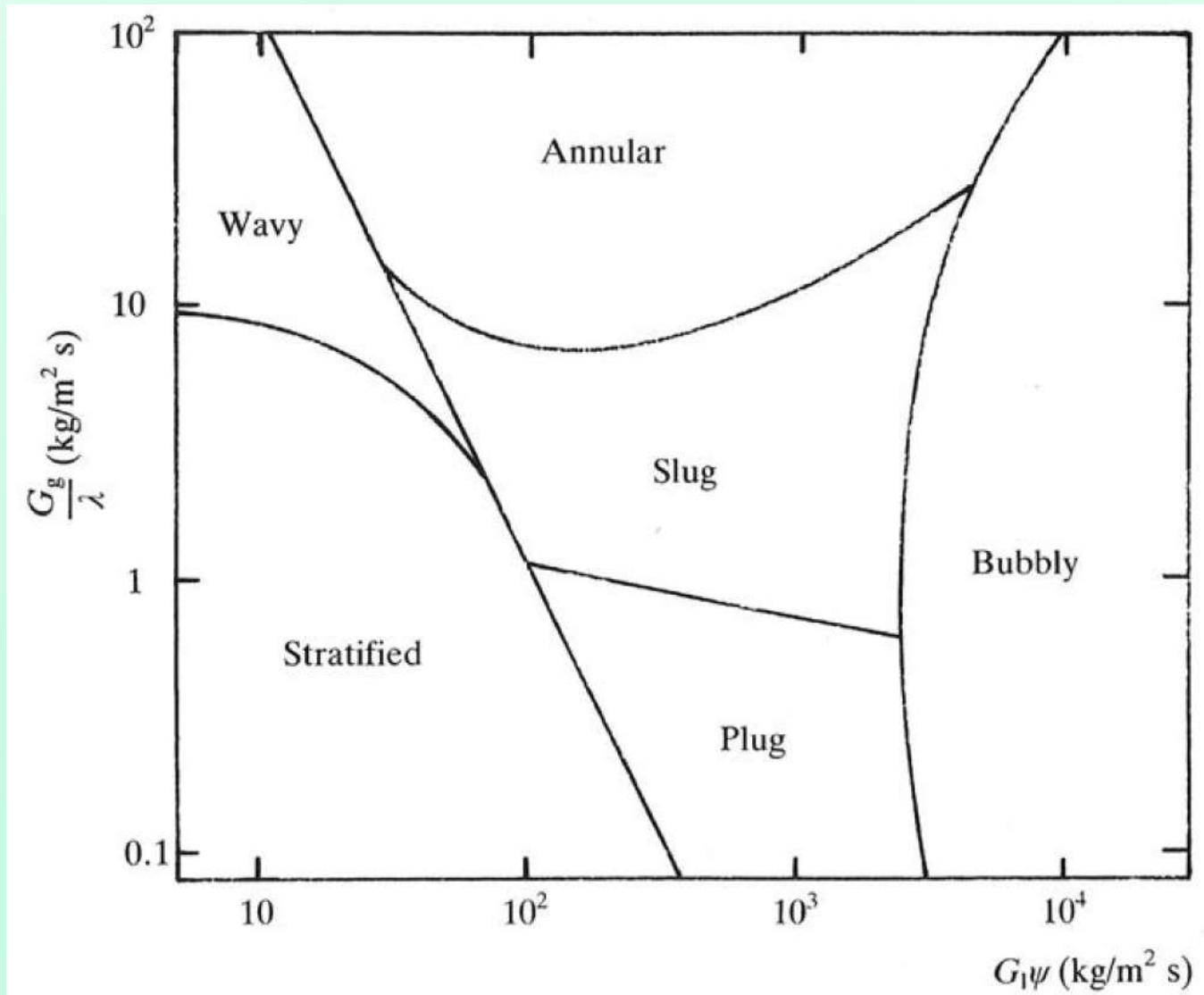
$$G_g = \frac{\text{Gas Mass Flow Rate}}{\text{Tube Crosssectional Area}}$$

$$\psi = \frac{\sigma_{water}}{\sigma} \left[ \frac{\mu_l}{\mu_{water}} \left( \frac{\rho_{water}}{\rho_l} \right)^2 \right]^{\frac{1}{3}}$$

$$G_l = \frac{\text{Liquid Mass Flow Rate}}{\text{Tube Crosssectional Area}}$$

- Properties of air and water are evaluated at standard atmospheric pressure and room temperature
- Standard dimensionless parameters  $\lambda$  and  $\psi$  take into account the variation in the properties of the fluid

# Horizontal, Co-current: Baker (1954)



Works for R12 in 8 mm diameter horizontal tube

# Taitel and Dukler Map, 1976

- Proposed in 1976 for horizontal flow in tube
- Originally developed for adiabatic flow with no phase change
- The map uses Martinelli parameter ( $X_{tt}$ ) the gas Froude number ( $Fr_G$ ) and the parameters  $T$  and  $K$

## Procedure

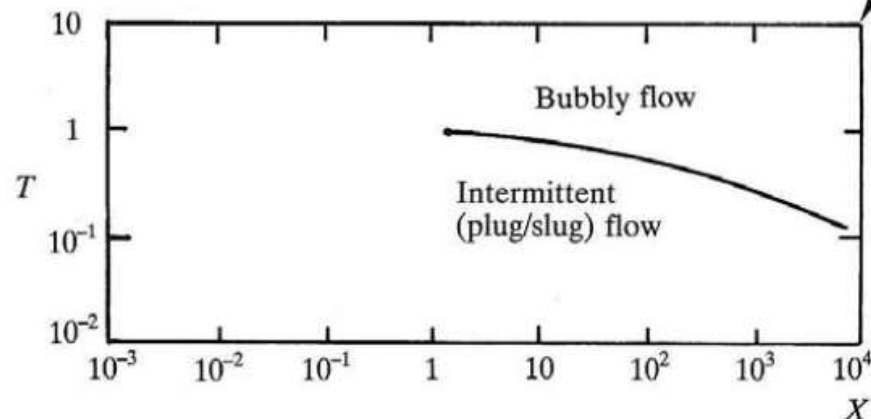
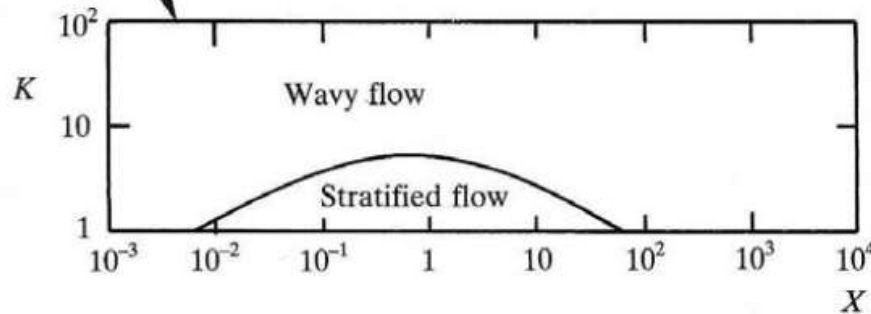
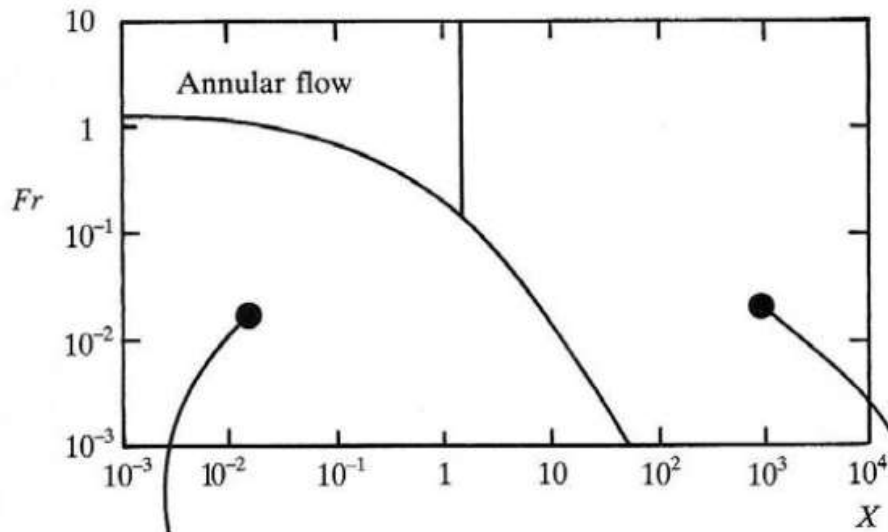
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- The map uses Martinelli parameter ( $X$ ) the gas Froude number ( $Fr_g$ ) and the parameters  $T$  and  $K$

## Procedure

- If  $(Fr_g, X)$  falls in **annular flow regime**, then the flow is
- If  $(Fr_g, X)$  falls in the lower left zone
  - Using  $(K, X)$ , identify **stratified-wavy or fully stratified**
- If  $(Fr_g, X)$  falls in the right zone
  - Using  $(T, X)$ , identify **bubbly or intermittent (plug/slug)**

The map was tested for condensation with water, methanol, propanol, R113, N-pentane in 24.4 mm tube



$$Fr_g = \frac{G_g}{[\rho_g(\rho_l - \rho_g)D_g]^{\frac{1}{2}}}$$

$$T = \left[ \frac{|(dp/dz)_L|}{g(\rho_l - \rho_g)} \right]^{\frac{1}{2}}$$

$$K = Fr_g Re_l^{\frac{1}{2}}$$

$$X = \left[ \frac{(dp/dz)_l}{(dp/dz)_g} \right]^{\frac{1}{2}}$$

$$Re_f = \frac{G_f D}{\mu_f}, f \text{ is either } g \text{ or } l$$

$$(dp/dz)_f = \frac{2f_f G_f^2}{\rho_f D}$$

$$f_f = \frac{16}{Re_f}, Re_f \leq 2000$$

$$f_f = \frac{0.079}{Re_f}, Re_f > 2000$$

## Problem: Flow Pattern in Vertical and Horizontal

Find the flow pattern when 4 kg/s of steam-water mixture of quality 20% at 20 bar flows in a 0.1 m circular tube.

$$\rho_l = 850 \text{ kg/m}^3, \rho_g = 10 \text{ kg/m}^3, \mu_l = 128 \times 10^{-6} \text{ Pa s}, \mu_g = 16 \times 10^{-6} \text{ Pa s}$$

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**Vertical Upflow:**

$$G = \frac{4}{\pi 0.1^2 / 4} = 509 \text{ kg/m}^2\text{s}$$

$$G_g = xG = 102 \text{ kg/m}^2\text{s}$$

$$G_l = (1 - x)G$$

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